# North Carolina Math 1 Standards

<table>
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<th>Number</th>
<th>Algebra</th>
<th>Functions</th>
<th>Geometry</th>
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<tbody>
<tr>
<td><strong>The real number system</strong> Extend the properties of exponents</td>
<td>Reasoning with equations and inequalities Understand solving equations as a process of reasoning and explain the reasoning</td>
<td>Overview</td>
<td>Expressing geometric properties with equations Use coordinates to prove simple geometric theorems algebraically</td>
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<tr>
<td><strong>Overview</strong></td>
<td>Solve equations and inequalities in one variable</td>
<td>NC.M1.A-REI.2</td>
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<tr>
<td>Write expressions in equivalent forms to solve problems</td>
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<td>NC.M1.A-REI.5</td>
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<tr>
<td><strong>Perform arithmetic operations on polynomials</strong> Perform arithmetic operations on polynomials</td>
<td>Represent and solve equations and inequalities graphically</td>
<td>NC.M1.A-REI.7</td>
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</tr>
<tr>
<td><strong>Creating Equations</strong> Create equations that describe numbers or relationships</td>
<td>NC.M1.F-BF.1a</td>
<td>Building Functions Build a function that models a relationship between two quantities</td>
<td>NC.M1.S-ID.1</td>
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<tr>
<td>NC.M1.A-CED.2</td>
<td>NC.M1.F-BF.2</td>
<td>Linear, Quadratics and Exponential Models Construct and compare linear and exponential models to solve problems</td>
<td>NC.M1.S-ID.3</td>
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</table>
Standards for Mathematical Practice

<table>
<thead>
<tr>
<th>Practice</th>
<th>Explanation</th>
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</thead>
<tbody>
<tr>
<td>1. Make sense of problems and persevere in solving them.</td>
<td>Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.</td>
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<tr>
<td>2. Reason abstractly and quantitatively.</td>
<td>Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.</td>
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<tr>
<td>3. Construct viable arguments and critique the reasoning of others.</td>
<td>Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.</td>
</tr>
</tbody>
</table>
4. Model with mathematics. Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically. Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision. Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure. Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see $7 \times 8$ equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as $2 \times 7$ and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic
expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers $x$ and $y$.

<table>
<thead>
<tr>
<th>8. Look for and express regularity in repeated reasoning.</th>
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<tbody>
<tr>
<td>Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.</td>
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## NC Math 1
### Unit 1: Equations & Introduction to Functions

<table>
<thead>
<tr>
<th>Learning Outcomes</th>
<th>Student Objectives</th>
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<tbody>
<tr>
<td>• Given an equation students will solve and justify their method and steps of solving.</td>
<td>• I will <strong>solve</strong> an equation or inequality and <strong>justify</strong> my steps.</td>
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<tr>
<td>• Students will be able to interpret key features of expressions, equations, graphs, tables, and verbal descriptions in context.</td>
<td><strong>NC.M1.A.REI.3, NC.M1.A.REI.1, NC.M1.A.REI.12</strong></td>
</tr>
<tr>
<td>• Create an equation or inequality and interpret reasonable solutions in context.</td>
<td>• I will be able to <strong>interpret</strong> key features and solutions.</td>
</tr>
<tr>
<td>• Given a formula students will solve for a specified variable.</td>
<td><strong>NC.M1.A-SSE.1a, NC.M1.F-IF.4</strong></td>
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<tr>
<td>• Given a function students will determine domain and range.</td>
<td>• I will be able to <strong>create</strong> an equation or inequality from a word problem.</td>
</tr>
<tr>
<td>• Given a function create an equation from various representations and use them to solve problems.</td>
<td>• I will <strong>manipulate</strong> a formula to <strong>solve</strong> for a specific variable.</td>
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<tr>
<td></td>
<td><strong>NC.M1.A-CED.1</strong></td>
</tr>
<tr>
<td></td>
<td>• I will <strong>manipulate</strong> a formula to <strong>solve</strong> for a specific variable.</td>
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<tr>
<td></td>
<td><strong>NC.M1.A-CED.4</strong></td>
</tr>
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</table>

RESEARCH BRIEF: **Unit 1: Equations & Introduction to Functions**

**Essential Questions:**

- How can data tables, graphs, and rules relating variables be used to answer questions about relationships between variables?
- How do dependent variables change as independent variables change?
- How can equations and inequalities be used to model real world situations?
- Given a function in function notation students will evaluate and interpret results in context.
- Understand what it takes to be a function in categorical, numerical, and graphical scenarios.
- Students should be able to understand functions as a correspondence between inputs and outputs.

- I will be able to recognize domain and range values in a function. \( \text{NC.M1.F-IF.1} \)
- I will evaluate a function for a given value. \( \text{NC.M1.F-IF.2} \)
- I will understand what a function is. \( \text{NC.M1.F-IF.1} \)
- I will identify the relationship between input and output. \( \text{NC.M1.F-IF.1} \)

# Standards Addressed in this Unit

## Construct expressions, equations, and inequalities from a given context and determine the appropriateness of the solution.

<table>
<thead>
<tr>
<th>OCS Priority Standard(s)</th>
<th>Supporting Standard(s)</th>
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<tbody>
<tr>
<td>\textbf{NC.M1.A-SSE.1a}: Interpret expressions that represent a quantity in terms of its context. Identify and interpret parts of a linear, exponential, or quadratic expression, including terms, factors, coefficients, and exponents. \textbf{NC.M1.A-CED.1}: Create equations that describe numbers or relationships. Create equations and inequalities in one variable that represent linear, exponential, and quadratic relationships and use them to solve problems.</td>
<td>\textbf{NC.M1.A-REI.3}: Solve linear equations and inequalities in one variable. \textbf{NC.M1.A-REI.1}: Understand solving equations as a process of reasoning and explain the reasoning. Justify a chosen solution method and each step of the solving process for linear and quadratic equations using mathematical reasoning. \textbf{NC.M1.A-REI.12}: Represent the solutions of a linear inequality or a system of linear inequalities graphically as a region of the plane. \textbf{NC.M1.A-CED.4}: Create equations that describe numbers or relationships. Solve for a quantity of interest in formulas used in science and mathematics using the same reasoning as in solving equations.</td>
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September 2017 Update - Feedback  
NC Math 1 Unit 1 Equations & Introductions to Functions  
Return to: Standards Addressed in this Unit
Distinguish key features of a function given multiple representations.

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<th>OCS Priority Standard(s)</th>
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<tr>
<td><strong>NC.M1.F-IF.4</strong>: Interpret functions that arise in applications in terms of the context. Interpret key features of graphs, tables, and verbal descriptions in context to describe functions that arise in applications relating two quantities, including: intercepts; intervals where the function is increasing, decreasing, positive, or negative; and maximums and minimums.</td>
<td><strong>NC.M1.F-IF.1</strong>: Understand the concept of a function and use function notation. Build an understanding that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range by recognizing that: ○ if ( f ) is a function and ( x ) is an element of its domain, then ( f(x) ) denotes the output of ( f ) corresponding to the input ( x ) ○ the graph of ( f ) is the graph of the equation ( y = f(x) ). <strong>NC.M1.F-IF.2</strong>: Understand the concepts of functions and use function notation. Use function notation to evaluate linear, quadratic, and exponential functions for inputs in their domains, and interpret statements that use function notation in terms of a context. <strong>NC.M1.F-IF.6</strong>: Interpret functions that arise in applications in terms of the context. Calculate and interpret the average rate of change over a specified interval for a function presented numerically, graphically, and/or symbolically.</td>
</tr>
</tbody>
</table>
NC.M1.A-SSE.1a
Interpret the structure of expressions.
Interpret expressions that represent a quantity in terms of its context.
  a. Identify and interpret parts of a linear, exponential, or quadratic expression, including terms, factors, coefficients, and exponents.

**Concepts and Skills**

<table>
<thead>
<tr>
<th>Pre-requisite</th>
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<tbody>
<tr>
<td>● Identify parts of an expression using precise vocabulary (6.EE.2b)</td>
</tr>
<tr>
<td>● Interpret numerical expressions written in scientific notation (8.EE.4)</td>
</tr>
<tr>
<td>● For linear and constant terms in functions, interpret the rate of change and the initial value (8.F.4)</td>
</tr>
</tbody>
</table>

**Connections**

| Interpreting changes in the parameters of a linear and exponential function in context (NC.M1.F-LE.5) |

**The Standards for Mathematical Practices**

**Implementing the Standards for Mathematical Practice**

<table>
<thead>
<tr>
<th>Connections</th>
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<tbody>
<tr>
<td>Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.</td>
</tr>
<tr>
<td>2 – Reason abstractly and quantitatively.</td>
</tr>
<tr>
<td>4 – Model with mathematics</td>
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<tr>
<td>7 – Look for and make use of structure.</td>
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</table>

**Disciplinary Literacy**

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary: Quadratic term, exponential term

**Mastering the Standard for this Unit**

**Comprehending the Standard**

This set of standards requires students:
  ● to write expressions in equivalent forms to reveal key quantities in terms of its context.
  ● to choose and use appropriate mathematics to analyze situations.

**Assessing for Understanding**

Students should recognize that in the expression $2x + 1$, “2” is the coefficient, “2” and “$x$” are factors, and “1” is a constant, as well as “$2x$” and “1” being terms of the binomial expression. Development and proper use of mathematical language is an important building block for future content. Using real-world context examples, the nature of algebraic expressions can be explored.

**Example:** The height (in feet) of a balloon filled with helium can be expressed by $5 + 6.3s$ where $s$ is the number of seconds since the balloon was released. Identify and interpret the terms and coefficients of the expression.
For this part of the standards, students recognize that the linear expression $mx + b$ has two terms, $m$ is a coefficient, and $b$ is a constant. Students extend beyond simplifying an expression and address interpretation of the components in an algebraic expression.
NC.M1.A-REI.3
_Solve equations and inequalities in one variable._
Solve linear equations and inequalities in one variable.

### Concepts and Skills

**Pre-requisite**
- Solving multi-step equations (8.EE.7)
- Solving two-step inequalities (7.EE.4)

**Connections**
- Create one variable linear equations and inequalities (NC.M1.A-CED.1)
- Justify a solution methods and the steps in the solving process (NC.M3.A-REI.1)
- Solve systems of linear equations (NC.M1.A-REI.6)

### The Standards for Mathematical Practices

**Implementing the Standards for Mathematical Practice**

**Connections**

_Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard._

**Disciplinary Literacy**

_As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication._

Students should be able to discuss their solution method and the steps in the solving process and should be able to interpret the solutions in context.

### Mastering the Standard for this Unit

**Comprehending the Standard**

Students are taught to solve multi-step equations in 8th grade. Students should become fluent solving multi-step equations in Math 1.

Students were taught to solve two-step inequalities in 7th grade. In Math 1 students extend this skill to multi-step inequalities.

This should be taught with the mathematical reasoning found in NC.M1.A-REI.1. Students should not be presented with a list steps to solve a linear equation/inequalities. Like many purely procedural practices, such steps are only effective for linear equations. It is more effective for students to be taught the mathematical reasoning for the solving process as these concepts can be applied to all types of equations.

**Assessing for Understanding**

Students should be able to solve multistep linear equations and inequalities.

**Example:** Solve:

- \( \frac{7}{3}y - 8 = 111 \)
- \( 3x - 2 > 9 + 5x \)
- \( \frac{3 + x}{7} = \frac{x - 9}{4} \)
- \( \frac{2}{3}x + 9 < 8 \left( \frac{1}{3}x - 2 \right) \)
- \( \frac{1}{5} \left( 10 - 20x \right) \leq -14 \)
NC.M1.A-REI.1
*Understand solving equations as a process of reasoning and explain the reasoning.*
Justify a chosen solution method and each step of the solving process for linear and quadratic equations using mathematical reasoning.

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<th>Concepts and Skills</th>
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<tbody>
<tr>
<td><strong>Pre-requisite</strong></td>
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<tr>
<td>● Students have been using properties of operations and equality throughout middle school. (6.EE.3, 7.EE.1, 7.EE.4). This is the first time that justification is required by a content standard.</td>
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<tr>
<td>● Solve multi-step equations (8.EE.7)</td>
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<tr>
<td><strong>Connections</strong></td>
</tr>
<tr>
<td>● Understand the relationship between factors of a quadratic equation and the solution of the equation (NC.M1.A-APR.3)</td>
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<tr>
<td>● Create and solve one variable linear and quadratic equations (NC.M1.A-CED.1)</td>
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<tr>
<td>● Solve for a quantity of interest in a formula (NC.M1.A-CED.4)</td>
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<table>
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<tr>
<td><strong>Implementing the Standards for Mathematical Practice</strong></td>
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<tr>
<td><strong>Connections</strong></td>
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<td>Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.</td>
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<tr>
<td>3 – Construct viable arguments and critique the reasoning of others</td>
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<th>Disciplinary Literacy</th>
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<tr>
<td>As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.</td>
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<tr>
<td>Students should be able to defend their method of solving an equation and each step of the solving process.</td>
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<tr>
<td>New Vocabulary: quadratic equation</td>
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<table>
<thead>
<tr>
<th>Mastering the Standard for this Unit</th>
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<tbody>
<tr>
<td><strong>Comprehending the Standard</strong></td>
</tr>
<tr>
<td>When solving equations, students will use the properties of equality to justify and explain each step obtained from the previous step, assuming the original equation has a solution, and develop an argument that justifies their method.</td>
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<tr>
<td>Properties of operations can be used to change expressions on either side of the equation to</td>
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<tr>
<td><strong>Assessing for Understanding</strong></td>
</tr>
<tr>
<td>Students should be able to justify a chosen solution method and justify each step in the process. This would be a good opportunity to discuss efficiency.</td>
</tr>
<tr>
<td><strong>Example:</strong> To the right are two methods to solve the same equation. Justify each step in the solving process. Which method do you prefer? Why?</td>
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</table>

*September 2017 Update - Feedback*
equivalent expressions. In the properties of equality, adding the same term to both sides of an equation or multiplying both sides by a non-zero constant produces an equation with the same solutions.

Students do not have to name the property, but can describe property using mathematical reasoning.

For example: Transforming $2x - 5 = 7$ to $2x = 12$ is possible because $5 = 5$, so adding the same quantity to both sides of an equation makes the resulting equation true as well.

### Method 1:

<table>
<thead>
<tr>
<th>Equation</th>
<th>Solution</th>
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<tbody>
<tr>
<td>$5(x + 3) - 3x = 55$</td>
<td>$5x + 15 - 3x = 55$</td>
</tr>
<tr>
<td>$5x + 15 - 3x = 55$</td>
<td>$2x + 15 = 55$</td>
</tr>
<tr>
<td>$2x + 15 - 15 = 55 - 15$</td>
<td>$2x = 40$</td>
</tr>
<tr>
<td>$2x = 40$</td>
<td>$\frac{2}{2} = \frac{40}{2}$</td>
</tr>
<tr>
<td>$x = 20$</td>
<td></td>
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</table>

### Method 2:

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>$5(x + 3) - 3x = 55$</td>
<td>$5(x + 3) - 3x = 55$</td>
</tr>
<tr>
<td>$5 - \frac{3}{5}x = \frac{5}{5}$</td>
<td>$x + 3 - \frac{3}{5}x = 11$</td>
</tr>
<tr>
<td>$x + 3 - \frac{3}{5}x = 11$</td>
<td>$\frac{2}{5}x + 3 = 11$</td>
</tr>
<tr>
<td>$\frac{2}{5}x + 3 - 3 = 11 - 3$</td>
<td>$\frac{2}{5}x = 8$</td>
</tr>
<tr>
<td>$\frac{2}{5}x = 8$</td>
<td>$\frac{5}{2}x = \frac{5}{2}(8)$</td>
</tr>
<tr>
<td>$x = 20$</td>
<td></td>
</tr>
</tbody>
</table>

Students should be able to critique the solving process of others, recognize incorrect steps and provide corrective action to the process.

### Example: The following is a student solution to the inequality $\frac{5}{18} \cdot \frac{x - 2}{9} \leq \frac{x - 4}{6}$.

a) There are two mathematical errors in this work. Identify at what step each mathematical error occurred and explain why it is mathematically incorrect.

b) How would you help the student understand his mistakes?

c) Solve the inequality correctly.

(https://www.illustrativemathematics.org/content-standards/HSA/REI/A/1/tasks/807)

Note: While this standard does not cover inequalities, this could be a good extension.
NC.M1.A-REI.12
*Represent and solve equations and inequalities graphically*

Represent the solutions of a linear inequality or a system of linear inequalities graphically as a region of the plane.

### Concepts and Skills

**Pre-requisite**
- Solve two-step linear inequalities (7.EE.4b)
- Solve linear inequalities in one variable (NC.M1.A-REI.3)
- Understand every point on a graph is a solution to its associated equation (NC.M1.A-REI.10)

**Connections**
- Create one variable linear inequalities and use the inequality to solve problems (NC.M1.A-CED.1)
- Create a system of linear inequalities to model a situation in context (NC.M1.A-CED.3)

### The Standards for Mathematical Practices

**Implementing the Standards for Mathematical Practice**

**Connections**

*Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.*

**Disciplinary Literacy**

*As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.*

Students should be able to explain the reasoning behind their graphical representation of an inequality or system of inequalities.

### Mastering the Standard for this Unit

**Comprehending the Standard**

Students should understand that since there is no way to list every solution to a linear inequality in two variables, the solutions must be represented graphically.

It is an American tradition to shade the region that represent the solutions of the inequality. In other countries, they shade regions of the plane that do not contain solutions, marking that region out. This results in an unmarked solution region making it easier to identify and work with points in the solution region. This means that it is important for students to understand what the shaded region represents according to the context of the problem.

**Assessing for Understanding**

Students should be able to represent and interpret solutions to one variable inequalities on a number line.

**Example:** Elvira, the cafeteria manager, has to be careful with her spending and manages the cafeteria so that they can serve the best food at the lowest cost. To do this, Elvira keeps good records and analyzes all of her budgets. Elvira’s cafeteria has those cute little cartons of milk that are typical of school lunch. The milk supplier charges $0.35 per carton of milk, in addition to a delivery charge of $75. What is the maximum number of milk cartons that Elvira can buy if she has budgeted $500 for milk?

a) Write and solve an inequality that models this situation, then graph the solution on a number line.

b) Describe in words the quantities that would work in this situation.

(www.mathematicsvisionproject.org)

**Example:** Fishing Adventures rents small fishing boats to tourists for day-long fishing trips. Each boat can hold at most eight people. Additionally, each boat can only carry 900 pounds of weight for safety reasons.

a) Let p represent the total number of people. Write an inequality to describe the number of people that a boat can hold. Draw a number line diagram that shows all possible solutions.
b) Let \( w \) represent the total weight of a group of people wishing to rent a boat. Write an inequality that describes all total weights allowed in a boat. Draw a number line diagram that shows all possible solutions.

(https://www.illustrativemathematics.org/content-standards/tasks/642)

**NC.M1.A-CED.1**

*Create equations that describe numbers or relationships.*

Create equations and inequalities in one variable that represent linear, exponential, and quadratic relationships and use them to solve problems.

<table>
<thead>
<tr>
<th>Concepts and Skills</th>
<th>The Standards for Mathematical Practices</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pre-requisite</strong></td>
<td><strong>Implementing the Standards for Mathematical Practice</strong></td>
</tr>
</tbody>
</table>
| • Create two-step linear equations and inequalities from a context (7.EE.4) | *Connections*  
  Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.  
  4 – Model with mathematics |
| **Connections**      | *Disciplinary Literacy*  
  As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.  
  Students should be able to describe the origins of created equations and inequalities and demonstrate its relation to the context.  
  New Vocabulary: exponential function, quadratic function |
| • Interpret parts of an expression in context (NC.M1.A-SSE.1a,b)  
• Justify a chosen solution method and each step of a that process (NC.M1.A-REI.1)  
• Solve linear, exponential and quadratic equations using tables and graphs (NC.M1.A-REI.11)  
• Represent the solutions of linear inequalities on a graph (NC.M1.A-REI.12) |  |

**Mastering the Standard for this Unit**

<table>
<thead>
<tr>
<th>Comprehending the Standard</th>
<th>Assessing for Understanding</th>
</tr>
</thead>
</table>
| Students create equations and inequalities in one-variable and use them to solve the problems. In Math I, focus on linear, quadratic, and exponential contextual situations that | Students should be able to create an equation from a function and use the equation to solve problems.  
**Example:** A government buys \( x \) fighter planes at \( z \) dollars each, and \( y \) tons of wheat at \( w \) dollars each. It spends a total of \( B \) dollars, where \( B = xz + yw \). In (a)–(c), write an equation whose solution is the given quantity. |
students can use to create equations and inequalities in one variable and use them to solve problems. It is also important to note that equations can also be created from an associated function. After the students have created an equation, they can use other representations to solve problems, such as graphs and tables.

Students in Math I are not responsible for interval notation as a solution. They are to write answers to these inequalities using inequality notation.

| Students should be able to create equations and inequalities from various representations, such as verbal descriptions, and use them to solve problems. Example: Mary and Jeff both have jobs at a baseball park selling bags of peanuts. They get paid $12 per game and $1.75 for each bag of peanuts they sell. Create equations, that when solved, would answer the following questions:
| a) How many bags of peanuts does Jeff need to sell to earn $54? | b) How much will Mary earn if she sells 70 bags of peanuts at a game? | c) How many bags of peanuts does Jeff need to sell to earn at least $68? |
| The number of tons of wheat the government can afford to buy if it spends a total of $100 million, wheat costs $300 per ton, and it must buy 5 fighter planes at $15 million each. |
| The price of fighter planes if the government bought 3 of them, in addition to 10,000 tons of wheat at $500 a ton, for a total of $50 million. |
| The price of a ton of wheat, given that a fighter plane costs 100,000 times as much as a ton of wheat, and that the government bought 20 fighter planes and 15,000 tons of wheat for a total cost of $90 million. |

(https://www.illustrativemathematics.org/content-standards/HSA/CED/A/1/tasks/580)

NC.M1.A-CED.4

Create equations that describe numbers or relationships.

Solve for a quantity of interest in formulas used in science and mathematics using the same reasoning as in solving equations.

<table>
<thead>
<tr>
<th>Concepts and Skills</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pre-requisite</strong></td>
</tr>
<tr>
<td>● Solve linear equations in one variable (8.EE.7 and NC.M1.A-REI.3)</td>
</tr>
<tr>
<td>● Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$ where $p$ is a positive rational number (8.EE.2)</td>
</tr>
<tr>
<td>● Justify a solution method and each step in the solving process (NC.M1.A-REI.1)</td>
</tr>
<tr>
<td><strong>Connections</strong></td>
</tr>
<tr>
<td>● Create an equation in two variables that represent a relationship between quantities (NC.M1.A-CED.2)</td>
</tr>
<tr>
<td>● Justify a solving method and each step in the solving process (NC. M1.A-REI.1)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>The Standards for Mathematical Practices</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Implementing the Standards for Mathematical Practice</strong></td>
</tr>
<tr>
<td><strong>Connections</strong></td>
</tr>
<tr>
<td>Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.</td>
</tr>
<tr>
<td>4 – Model with mathematics</td>
</tr>
<tr>
<td>7 – Look for and make use of structure</td>
</tr>
</tbody>
</table>

**Disciplinary Literacy**

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication. Students should be able to justify the steps in their solving process.
<table>
<thead>
<tr>
<th>Comprehending the Standard</th>
<th>Assessing for Understanding</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students should be able to solve an equation for a given variable. In Math 1, focus on real mathematical and scientific formulas. This may be a good opportunity to talk with the science teachers and ask them for formulas that they use often. This standard also covers solve for variables in mathematical forms as well as formulas. (Students are not expected to write linear equation into “proper” standard form.)</td>
<td>Students should be able to solve for variables in mathematical forms as well as formulas. <strong>Example:</strong> Solve ((y - y_1) = m(x - x_1)) for (m).</td>
</tr>
</tbody>
</table>

Students should be able to solve for variable in science and math formula.  
**Example:** (NCDPI Math I released EOC #18) Energy and mass are related by the formula \(E = mc^2\).  
- \(m\) is the mass of the object  
- \(c\) is the speed of light  

Which equation finds \(m\), given \(E\) and \(c\)?  

A) \(m = E - c^2\)  
B) \(m = Ec^2\)  
C) \(m = \frac{c^2}{E}\)  
D) \(m = \frac{E}{c^2}\)  

**Example:** In each of the equations below, rewrite the equation, solving for the indicated variable.  
a) If \(F\) denotes a temperature in degrees Fahrenheit and \(C\) is the same temperature measured in degrees Celsius, then \(F\) and \(C\) are related by the equation \(F = 95C + 32\). Rewrite this equation to solve for \(C\) in terms of \(F\).  
b) The surface area \(S\) of a sphere of radius \(r\) is given by \(S = 4\pi r^2\). Solve for \(r\) in terms of \(S\).  
(https://www.illustrativemathematics.org/content-standards/HSA/CED/A/4/tasks/1828)  

**Example:** The equation for an object that is launched from the ground is given by \(h(t) = -16t^2 + v_0t\) where \(h\) is the height, \(t\) is the time, and \(v_0\) is the initial velocity. What is the initial velocity of an object that is one-hundred feet off the ground four seconds after it is launched?
NC.M1.F-IF.2

Understand the concept of a function and use function notation.

Use function notation to evaluate linear, quadratic, and exponential functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

The Standards for Mathematical Practices
Implementing the Standards for Mathematical Practice

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to discuss the domain, range, input, output and the relationship between the variables of a function in context.

New Vocabulary: exponential function, quadratic function

<table>
<thead>
<tr>
<th>Pre-requisite</th>
</tr>
</thead>
<tbody>
<tr>
<td>Use substitution to determine if a number if a solution (6.EE.5)</td>
</tr>
<tr>
<td>Interpret parts of expressions in context (NC.M1.A-SSE.1a, NC.M1.A-SSE.1b)</td>
</tr>
<tr>
<td>Every point on the graph of an equation is a solution to the equation (NC.M1.A-REI.10)</td>
</tr>
<tr>
<td>Define a function and use functions notation (NC.M1.F-IF.1)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Connections</th>
</tr>
</thead>
<tbody>
<tr>
<td>Creating and solving one variable equations (NC.M1.A-CED.1)</td>
</tr>
<tr>
<td>Creating and graphing two variable equations (NC.M1.A-CED.2)</td>
</tr>
<tr>
<td>Every point on the graph of an equation is a solution to the equation (NC.M1.A-REI.10)</td>
</tr>
<tr>
<td>Comparing the end behavior of functions (NC.M1.F-LE.3)</td>
</tr>
</tbody>
</table>

### Mastering the Standard for this Unit

<table>
<thead>
<tr>
<th>Comprehending the Standard</th>
<th>Assessing for Understanding</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students should be fluent in using function notation to evaluate a linear, quadratic, and exponential function. Students should be able to interpret statements in function notation in contextual situations.</td>
<td>Students should be able to use evaluate functions written in function notation. Example: Evaluate $f(2)$ for the function $f(x) = 5(x - 3) + 17$ Evaluate $f(2)$ for the function $f(x) = 1200(1 + .04)^x$ Evaluate $f(2)$ for the function $f(x) = 3x^2 + 2x - 5$</td>
</tr>
</tbody>
</table>
Students should be able to evaluate functions and interpret the result in a context.

**Example:** You placed a yam in the oven and, after 45 minutes, you take it out. Let $f$ be the function that assigns to each minute after you placed the yam in the oven, its temperature in degrees Fahrenheit. Write a sentence for each of the following to explain what it means in everyday language.

- a) $f(0) = 65$
- b) $f(5) < f(10)$
- c) $f(40) = f(45)$
- d) $f(45) > f(60)$  

([https://www.illustrativemathematics.org/content-standards/HSF/IF/A/2/tasks/625](https://www.illustrativemathematics.org/content-standards/HSF/IF/A/2/tasks/625))

**Example:** The rule $f(x) = 50(0.85)^x$ represents the amount of a drug in milligrams, $f(x)$, which remains in the bloodstream after $x$ hours. Evaluate and interpret each of the following:

- a) $f(0)$
- b) $f(2) = k \cdot f(1)$. What is the value of $k$?

**Example:** Suppose that the function $f(x) = 2x + 12$ represents the cost to rent $x$ movies a month from an internet movie club. Makayla now has $10. How many more dollars does Makayla need to rent 7 movies next month?  

(NCDPI Math 1 released EOC #12)

**Example:** Let $f(t)$ be the number of people, in millions, who own cell phones $t$ years after 1990. Explain the meaning of the following statements.

- a) $f(10) = 100.3$
- b) $f(a) = 20$
- c) $f(20) = b$
- d) $n = f(t)$

([https://www.illustrativemathematics.org/content-standards/HSF/IF/A/2/tasks/634](https://www.illustrativemathematics.org/content-standards/HSF/IF/A/2/tasks/634))
NC.M1.F-IF.1
Understand the concept of a function and use function notation.

Build an understanding that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range by recognizing that:

- if \( f \) is a function and \( x \) is an element of its domain, then \( f(x) \) denotes the output of \( f \) corresponding to the input \( x \).
- the graph of \( f \) is the graph of the equation \( y = f(x) \).

### Concepts and Skills

<table>
<thead>
<tr>
<th>Pre-requisite</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Understand that a function is a rule that assigns to each input exactly one output (8.F.1)</td>
</tr>
<tr>
<td>- Every point on the graph of an equation is a solution to the equation (NC.M1.A-REI.10)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Connections</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Create and graph two variable equations (NC.M1.A-CED.2)</td>
</tr>
<tr>
<td>- All other function standards</td>
</tr>
</tbody>
</table>

### The Standards for Mathematical Practices

#### Implementing the Standards for Mathematical Practice

#### Connections

*Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.*

#### Disciplinary Literacy

*As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.*

Students should be able to accurate describe a function in their own terms.

New Vocabulary: notation
Comprehending the Standard
Students should understand the definition of a function. It is deeper than just "x" cannot repeat or the vertical line test. Students should understand what it takes to be a function in categorical, numerical, and graphical scenarios.

In 8th grade, students studied the definition of a function. In Math 1, function notation is introduced. While this standard places a focus of the definition of a function on the correspondence of input and output values, a function can also be defined by how one variable changes in relation to another variable. This view of a function is highlighted in other standards throughout Math 1 when students are asked to identify, interpret, and use the rate of change. For this unit, domain and range may be given in inequality notation. Students in Math I are not responsible for interval or set notation as a solution. They are to write answers to these inequalities using inequality notation.

Assessing for Understanding
Students should be able to understand functions in categorical scenarios.

**Example:** A certain business keeps a database of information about its customers.

a) Let $C$ be the rule which assigns to each customer shown in the table his or her home phone number. Is $C$ a function? Explain your reasoning.
b) Let $P$ be the rule which assigns to each phone number in the table above, the customer name(s) associated with it. Is $P$ a function? Explain your reasoning.
c) Explain why a business would want to use a person's social security number as a way to identify a particular customer instead of their phone number.

(https://www.illustrativemathematics.org/content-standards/HSF/IF/A/1/tasks/624)

Students should be able to understand functions as a correspondence between inputs and outputs.

**Example:** A pack of pencils cost $0.75. If $n$ number of packs are purchased, then the total purchase price is represented by the function $t(n) = 0.75n$.

a) Explain why $t$ is a function.
b) What is a reasonable domain and range for the function $t$?

(https://www.illustrativemathematics.org/content-standards/HSF/IF/A/1/tasks/630)

<table>
<thead>
<tr>
<th>Customer Name</th>
<th>Home Phone Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heather Baker</td>
<td>3105100091</td>
</tr>
<tr>
<td>Mike London</td>
<td>3105200256</td>
</tr>
<tr>
<td>Sue Green</td>
<td>3234132598</td>
</tr>
<tr>
<td>Bruce Swift</td>
<td>3234132598</td>
</tr>
<tr>
<td>Michelle Metz</td>
<td>2138061124</td>
</tr>
</tbody>
</table>
NC.M1.F-IF.4

Interpret functions that arise in applications in terms of the context.

Interpret key features of graphs, tables, and verbal descriptions in context to describe functions that arise in applications relating two quantities, including: intercepts; intervals where the function is increasing, decreasing, positive, or negative; and maximums and minimums.

### Concepts and Skills

**Pre-requisite**
- Describe quantitatively the functional relationship between two quantities by analyzing a graph (8.F.5)
- Define a function and use functions notation (NC.M1.F-IF.1)
- Evaluating functions (NC.M1.F-IF.2)

**Connections**
- Interpret parts of expressions in context (NC.M1.A-SSE.1a, NC.M1.A-SSE.1b)
- Relate domain and range of a function to its graph (NC.M1.F-IF.5)
- Calculate the average rate of change (NC.M1.F-IF.6)
- Use equivalent forms of quadratic and exponential function to reveal key features (NC.M1.F-IF.8a, NC.M1.F-IF.8b)
- Compare key features of two functions in different representations (NC.M1.F-IF.9)
- Identify situations that can be modeled with linear and exponential functions (NC.M1.F-LE.1)

### The Standards for Mathematical Practices

**Implementing the Standards for Mathematical Practice**

**Connections**

*Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.*

4 – Model with mathematics

**Disciplinary Literacy**

*As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.*

Students should be able to justify their identification of key features and interpret those key features in context.

New Vocabulary: maximum, minimum

### Mastering the Standard for this Unit

<table>
<thead>
<tr>
<th>Comprehending the Standard</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Students should understand the key features of any contextual situation. For example, plots over time represent functions as do some scatterplots. These are often functions that “tell a story” hence the portion of the standard that has students sketching graphs given a verbal description.</td>
<td>Students should be able to identify and interpret key features of functions. <strong>Example:</strong> An epidemic of influenza spreads through a city. The figure below is the graph of $I = f(w)$, where $I$ is the number of individuals (in thousands) infected $w$ weeks after the epidemic begins.</td>
</tr>
</tbody>
</table>
description. Students should have experience with a wide variety of these types of functions and be flexible in thinking about functions and key features using tables, graphs, and verbal descriptions.

Students should understand the concept behind the key features (intercepts, increasing/decreasing, positive/negative, and maximum/minimum) for any given graph, not just “function families”. This means that students should be asked to work with graphical and tabular representations of functions that the student could not solve or manipulation algebraically.

By contrast, NC.M1.F-IF.7, has students work with specific functions in which students have the ability to use algebraic manipulation to identify additional key features.

<table>
<thead>
<tr>
<th>Time (minutes)</th>
<th>Distance (feet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>30</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
</tr>
<tr>
<td>4</td>
<td>25</td>
</tr>
<tr>
<td>5</td>
<td>50</td>
</tr>
</tbody>
</table>

Example: The figure shows the graph of $T$, the temperature (in degrees Fahrenheit) over one particular 20-hour period in Santa Elena as a function of time $t$.

a) Estimate $T(14)$.
b) If $t = 0$ corresponds to midnight, interpret what we mean by $T(14)$ in words.
c) Estimate the highest temperature during this period from the graph.
d) When was the temperature decreasing?
e) If Anya wants to go for a two-hour hike and return before the temperature gets over 80 degrees, when should she leave?

Example: Eliana observed her dog, Lola, running around the yard and recorded the time and distance that Lola was away from her dog house in the table below.

a) Sketch a graph of Lola’s play time away from her dog house.
b) Describe what is happening between minutes 2 & 3.

d) An equation for the function used to plot the image above is $f(w) = 6w(1.3)^w - w$. Use the graph to estimate the solution of the inequality $6w(1.3)^w - w \geq 6$. Explain what the solution means in terms of the epidemic. (This would make a great Honors level extension to this standard) (https://www.illustrativemathematics.org/content-standards/HSF/IF/B/4/tasks/637)

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Example: Eliana observed her dog, Lola, running around the yard and recorded the time and distance that Lola was away from her dog house in the table below.

a) Sketch a graph of Lola’s play time away from her dog house.
b) Describe what is happening between minutes 2 & 3.
NC.M1.F-IF.6

*Interpret functions that arise in applications in terms of the context.*

Calculate and interpret the average rate of change over a specified interval for a function presented numerically, graphically, and/or symbolically.

**Concepts and Skills**

**Pre-requisite**
- Determine and interpret the rate of change of a linear function (8.F.4)
- Describe qualitatively the functional relationship between two quantities and sketch a graph from a verbal description (8.F.5)

**Connections**
- Interpret key features of graphs and tables (NC.M1.F-IF.4)
- Analyze linear, quadratic and exponential functions by generating different representations (NC.M1.F-IF.7)

**The Standards for Mathematical Practices**

**Implementing the Standards for Mathematical Practice**

**Connections**

*Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.*

4 – Model with mathematics

**Disciplinary Literacy**

*As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.*

New Vocabulary: average rate of change

**Mastering the Standard in this Unit**

**Comprehending the Standard**

Students calculate the average rate of change of a function given a graph, table, and/or equation.

The average rate of change of a function \( y = f(x) \) over an interval \( a \leq x \leq b \) is \( \frac{\text{change in } y}{\text{change in } x} \)

This standard is more than just slope. It is asking students to find the average rate of change of any function over any given interval. Be sure to include multiple representations (numerically, graphically, or symbolically) of functions for students to work with.

**Assessing for Understanding**

Students should be able to find the average rate of change over a specified interval.

**Example:** Find the average rate of change of each of the following functions over the interval \( 1 \leq x \leq 5 \).

a) \( f(x) = 3x - 7 \)

b) \( g(x) = x^2 + 2x - 5 \)

c) \( h(x) = 3(2)^x \)

**Example:** The table below shows the average weight of a type of plankton after several weeks.

<table>
<thead>
<tr>
<th>Time (weeks)</th>
<th>Weight (ounces)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>0.04</td>
</tr>
<tr>
<td>9</td>
<td>0.07</td>
</tr>
<tr>
<td>10</td>
<td>0.14</td>
</tr>
<tr>
<td>11</td>
<td>0.25</td>
</tr>
<tr>
<td>12</td>
<td>0.49</td>
</tr>
</tbody>
</table>

(NCDPI Math 1 released EOC #21)

September 2017 Update - Feedback

NC Math 1 Unit 1 Equations & Introductions to Functions

Return to: Standards Addressed in this Unit
It is an important connection for further courses that students recognize that linear functions have consistent average rate of change over any interval, while functions like quadratics and exponentials do not have constant rates of change due to their curvature.

It is an important connection for further courses that students recognize that linear functions have consistent average rate of change over any interval, while functions like quadratics and exponentials do not have constant rates of change due to their curvature.

What is the average rate of change in weight of the plankton from week 8 to week 12?

a) 0.0265 ounce per week  
   b) 0.0375 ounce per week  
   c) 0.055 ounce per week  
   d) 0.1125 ounce per week

**Example:** The table below shows the temperature, $T$, in Tucson, Arizona $t$ hours after midnight. When does the temperature decrease the fastest: between midnight and 3 a.m. or between 3 a.m. and 4 a.m.?

<table>
<thead>
<tr>
<th>$t$ (hours after midnight)</th>
<th>0</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$ (temp. in °F)</td>
<td>85</td>
<td>76</td>
<td>70</td>
</tr>
</tbody>
</table>

([https://www.illustrativemathematics.org/content-standards/HSF/IF/B/6/tasks/1500](https://www.illustrativemathematics.org/content-standards/HSF/IF/B/6/tasks/1500))

**Example:** You are a marine biologist working for the Environmental Protection Agency (EPA). You are concerned that the rare coral mathemafish population is being threatened by an invasive species known as the fluted dropout shark. The fluted dropout shark is known for decimating whole schools of fish. Using a catch-tag-release method, you collected the following population data over the last year.

<table>
<thead>
<tr>
<th># months since 1st measurement</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathemafish population</td>
<td>480</td>
<td>472</td>
<td>417</td>
<td>318</td>
<td>240</td>
<td>152</td>
<td>103</td>
<td>84</td>
<td>47</td>
<td>32</td>
<td>24</td>
<td>29</td>
<td>46</td>
</tr>
</tbody>
</table>
Through intervention, the EPA was able to reduce the dropout population and slow the decimation of the mathemafish population. Your boss asks you to summarize the effects of the EPA’s intervention plan in order to validate funding for your project.

What to include in your summary report:

a) Calculate the average rate of change of the mathemafish population over specific intervals. Indicate how and why you chose the intervals you chose.

b) When was the population decreasing the fastest?

c) During what month did you notice the largest effects of the EPA intervention?

d) Explain the overall effects of the intervention.

e) Remember to justify all your conclusions using supporting evidence.

(https://www.illustrativemathematics.org/content-standards/HSF/IF/B/6/tasks/686)
## NC Math 1
### Unit 2: Linear Functions

#### 15 Days Block Schedule

<table>
<thead>
<tr>
<th>RESEARCH BRIEF: Unit 2: Linear Functions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Essential Questions:</strong></td>
</tr>
<tr>
<td>● How can data tables, graphs, and rules relating variables be used to answer questions about relationships between variables?</td>
</tr>
<tr>
<td>● How do dependent variables change as independent variables change?</td>
</tr>
<tr>
<td>● How can equations and inequalities be used to model real world situations?</td>
</tr>
<tr>
<td>● How can geometric properties in the coordinate plane be used to classify figures?</td>
</tr>
</tbody>
</table>

#### Learning Outcomes

<table>
<thead>
<tr>
<th>Learning Outcomes</th>
<th>Student Objectives</th>
</tr>
</thead>
<tbody>
<tr>
<td>● Students will describe the relationships among the graph, symbolic rule, table of values, and related situation for a linear function.</td>
<td>● I will interpret the relationship of a linear function from various representations.</td>
</tr>
<tr>
<td>● Given a linear function, students will interpret the meaning of the slope and y-intercept of the function in context of the scenario.</td>
<td>● I will interpret the meaning of the slope and y-intercept given a linear scenario. NC.M1.A-SSE.1a, NC.M1.A-SSE.1b</td>
</tr>
<tr>
<td>● Students will interpret the meaning of the slope and y-intercept of the graph of a linear function in context.</td>
<td>● I will interpret the meaning of the slope and y-intercept from a graph of a scenario. NC.M1.A-SSE.1a, NC.M1.A-SSE.1b</td>
</tr>
<tr>
<td>● Given a graph, students will write the equation for a linear function.</td>
<td>● I will create a graph of a linear function. NC.M1.A-CED.2</td>
</tr>
<tr>
<td>● Given two points on a line, students will write the equation for</td>
<td>● I will create a linear equation given a graph, table of values, or verbal description. NC.M1.A-CED.2, NC.M1.F-BF.1a</td>
</tr>
</tbody>
</table>

**September 2017 Update - Feedback**
the linear function passing through the points.

- Given a table of values, students will write the equation for the linear function associated with the table.
- Students will determine the domain and range of linear functions and interpret them in context of the scenario.
- Students will use linear functions to answer questions about the situations that they describe.
- Students will use a linear model to predict the value of one variable given the value of the other and describe the rate of change in one variable as the other increases in a meaningful way.
- Students will use technology, graphs and table to approximate the solutions to linear equations.
- Students will use a graphing calculator or computer software to find the linear regression model for a set of data.
- Students will use technology to determine the correlation coefficient and interpret it as a measure of the strength and direction of a linear relationship.
- Students will write linear inequalities to represent scenarios.
- Students will estimate solutions to linear equations and inequalities by inspecting appropriate graphs and tables and interpret the meaning of the solutions in context.
- Students will solve and graph solutions of linear inequalities and interpret the meaning of the solution in context.
- Given an arithmetic sequence, students will write the recursive form for the sequence.
- Given an arithmetic sequence, students will write the explicit form for the sequence.
- Students will recognize that an arithmetic sequence is a linear function whose domain is a subset of the integers.
- Students will recognize that the terms of an arithmetic sequence are a subset of the range of a linear function.
- Students will use coordinates to find the midpoint or endpoint of a line segment.

- I will identify the domain and range of a linear function in context. NC.M1.F-IF.5, NC.M1.F-IF.7
- I will predict values based on a linear equation from a scenario.
- I will use technology, tables and graphs to approximate the solution to a linear equation. NC.M1.A-REI.11
- Given the value of one variable, I will predict the value of the other variable and interpret the rate of change in context. NC.M1.S-ID.6a, NC.M1.S-ID.7
- I will use technology to find the linear regression from a set of data. NC.M1.S-ID.6a
- I will create and solve linear inequalities from a scenario.
- I will estimate solutions from an equation or inequality by analyzing graphs and tables and interpret the meaning of the solution.
- I will solve and graph linear inequalities.
- I will interpret the meaning of the solutions to a linear inequality in context.
- I will write an arithmetic sequence in recursive form. NC.M1.F-BF.1a
- I will write an arithmetic sequence in explicit form. NC.M1.F-BF.1a
- I will understand the relationship between an arithmetic sequence and a linear function, including domain and range.
- I will use coordinates to determine the midpoint or endpoint of a line segment. NC.M1.G-GPE.6
- I will determine lines are parallel, perpendicular, or neither as defined by the slope rules. NC.M1.G-GPE.5
- I will write the equation of a line parallel or perpendicular to a given line through a given point. NC.M1.G-GPE.5
- I will compute the perimeter and area of polygons using the Pythagorean Theorem. NC.M1.G-GPE.4
● Students will use coordinates to prove the slope criteria for parallel and perpendicular lines.
● Students will find the equation of a line parallel or perpendicular to a given line and passing through a specified point.
● Students will use coordinates to compute the perimeter and area of triangles, rectangles, and polygons.
● Students will verify algebraically that a given set of points produces a particular type of triangle or quadrilateral.

I will **verify/classify** the type of quadrilateral given four points. \[\text{NC.M1.G-GPE.4}\]

### Standards Addressed in this Unit

**Identify, create, and graph linear equations and inequalities and interpret their key features.**

<table>
<thead>
<tr>
<th>OCS Priority Standard(s)</th>
<th>Supporting Standard(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>NC.M1.A-SSE.1a</strong>: Identify and interpret parts of a linear, exponential, or quadratic expression, including terms, factors, coefficients, and exponents.</td>
<td><strong>NC.M1.A-REI.12</strong>: Represent the solutions of a linear inequality or a system of linear inequalities graphically as a region of the plane.</td>
</tr>
<tr>
<td><strong>NC.M1.A-SSE.1b</strong>: Interpret a linear, exponential, or quadratic expression made of multiple parts as a combination of entities to give meaning to an expression.</td>
<td><strong>NC.M1.F-BF.1a</strong>: Build a function that models a relationship between two quantities. Write a function that describes a relationship between two quantities. Build linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two ordered pairs (include reading these from a table).</td>
</tr>
<tr>
<td><strong>NC.M1.A-CED.1</strong>: Create equations and inequalities in one variable that represent linear, exponential, and quadratic relationships and use them to solve problems.</td>
<td><strong>NC.M1.A-REI.11</strong>: Build an understanding of why the x-coordinates of the points where the graphs of two linear, exponential, or quadratic equations and intersect are the solutions of the equation and approximate solutions using a graphing technology or successive approximations with a table of values.</td>
</tr>
<tr>
<td><strong>NC.M1.A-CED.2</strong>: Create and graph equations in two variables to represent linear, exponential, and quadratic relationships between quantities.</td>
<td><strong>NC.M1.A-REI.10</strong>: Represent and solve equations and inequalities graphically. Understand that the graph of a two variable equation represents the set of all solutions to the equation.</td>
</tr>
</tbody>
</table>
**NC.M1.G-GPE.5:** Use coordinates to prove simple geometric theorems algebraically. Use coordinates to prove the slope criteria for parallel and perpendicular lines and use them to solve problems. Determine if two lines are parallel, perpendicular, or neither. Find the equation of a line parallel or perpendicular to a given line that passes through a given point.

---

Determine the explicit and recursive formula for given arithmetic sequence.

<table>
<thead>
<tr>
<th>OCS Priority Standard(s)</th>
<th>Supporting Standard(s)</th>
</tr>
</thead>
</table>
| **NC.M1.F-IF.3:** Understand the concept of a function and use function notation. Recognize that recursively and explicitly defined sequences are functions whose domain is a subset of the integers, the terms of an arithmetic sequence are a subset of the range of a linear function, and the terms of a geometric sequence are a subset of the range of an exponential function.  
**NC.M1.F-BF.2:** Build a function that models a relationship between two quantities. Translate between explicit and recursive forms of arithmetic and geometric sequences and use both to model situations.  
**NC.M1.A-REI.1:** Understand solving equations as a process of reasoning and explain the reasoning. Justify a chosen solution method and each step of the solving process for linear and quadratic equations using mathematical reasoning. |
Understand and compare key features of linear functions.

<table>
<thead>
<tr>
<th>OCS Priority Standard(s)</th>
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</tr>
</thead>
<tbody>
<tr>
<td><strong>NC.M1.F-IF.7:</strong> Analyze functions using different representations. Analyze linear, exponential, and quadratic functions by generating different representations, by hand in simple cases and using technology for more complicated cases, to show key features, including: domain and range; rate of change; intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximums and minimums; and end behavior. <strong>NC.M1.F-IF.9:</strong> Compare key features of two functions (linear, quadratic, or exponential) each with a different representation (symbolically, graphically, numerically in tables, or by verbal descriptions).</td>
<td><strong>NC.M1.F-LE.5:</strong> Interpret expressions for functions in terms of the situation they model. Interpret the parameters $a$ and $b$ in a linear function $f(x)=ax+b$ or an exponential function $g(x)=ab^x$ in terms of a context. <strong>NC.M1.F-IF.5:</strong> Interpret functions that arise in applications in terms of the context. Interpret a function in terms of the context by relating its domain and range to its graph and, where applicable, to the quantitative relationship it describes. <strong>NC.M1.S-ID.9:</strong> Interpret linear models. Distinguish between association and causation.</td>
</tr>
</tbody>
</table>

Assess the line of best fit for a given set of data by using the correlation coefficient, residuals, and the least squares regression line.

<table>
<thead>
<tr>
<th>OCS Priority Standard(s)</th>
<th>Supporting Standard(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>NC.M1.S-ID.6a:</strong> Summarize, represent, and interpret data on two categorical and quantitative variables. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Fit a least squares regression line to linear data using technology. Use the fitted function to solve problems. <strong>NC.M1.S-ID.6b:</strong> Summarize, represent, and interpret data on two categorical and quantitative variables. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Assess the fit of a linear function by analyzing residuals.</td>
<td><strong>NC.M1.S-ID.7:</strong> Interpret linear models. Interpret in context the rate of change and the intercept of a linear model. Use the linear model to interpolate and extrapolate predicted values. Assess the validity of a predicted value. <strong>NC.M1.S-ID.8:</strong> Interpret linear models. Analyze patterns and describe relationships between two variables in context. Using technology, determine the correlation coefficient of bivariate data and interpret it as a measure of the strength and direction of a linear relationship. Use a scatter plot, correlation coefficient, and a residual plot to determine the appropriateness of using a linear function to model a relationship between two variables.</td>
</tr>
</tbody>
</table>
Use geometric properties to classify & prove figures in the coordinate plane.

<table>
<thead>
<tr>
<th>OCS Priority Standard(s)</th>
<th>Supporting Standard(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>NC.M1.G-GPE.4</strong>: Use coordinates to prove simple geometric theorems algebraically. Use coordinates to solve geometric problems involving polygons algebraically. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles. Use coordinates to verify algebraically that a given set of points produces a particular type of triangle or quadrilateral.</td>
<td></td>
</tr>
<tr>
<td><strong>NC.M1.G-GPE.5</strong>: Use coordinates to prove simple geometric theorems algebraically. Use coordinates to prove the slope criteria for parallel and perpendicular lines and use them to solve problems. Determine if two lines are parallel, perpendicular, or neither. Find the equation of a line parallel or perpendicular to a given line that passes through a given point.</td>
<td></td>
</tr>
<tr>
<td><strong>NC.M1.G-GPE.6</strong>: Use coordinates to prove simple geometric theorems algebraically. Use coordinates to find the midpoint or endpoint of a line segment.</td>
<td></td>
</tr>
</tbody>
</table>
NC.M1.A-SSE.1a

*Interpret the structure of expressions.*

Interpret expressions that represent a quantity in terms of its context.

a. Identify and interpret parts of a linear, exponential, or quadratic expression, including terms, factors, coefficients, and exponents.

### Concepts and Skills

<table>
<thead>
<tr>
<th>Pre-requisite</th>
</tr>
</thead>
<tbody>
<tr>
<td>● Identify parts of an expression using precise vocabulary (6.EE.2b)</td>
</tr>
<tr>
<td>● Interpret numerical expressions written in scientific notation (8.EE.4)</td>
</tr>
<tr>
<td>● For linear and constant terms in functions, interpret the rate of change and the initial value (8.F.4)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Connections</th>
</tr>
</thead>
<tbody>
<tr>
<td>● Interpreting changes in the parameters of a linear and exponential function in context (NC.M1.F-LE.5)</td>
</tr>
</tbody>
</table>

### The Standards for Mathematical Practices

#### Implementing the Standards for Mathematical Practice

**Connections**

*Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.*

- 2 – Reason abstractly and quantitatively.
- 4 – Model with mathematics
- 7 – Look for and make use of structure.

**Disciplinary Literacy**

*As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.*

New Vocabulary: Quadratic term, exponential term

### Mastering the Standard in This Unit

<table>
<thead>
<tr>
<th>Comprehending the Standard</th>
<th>Assessing for Understanding</th>
</tr>
</thead>
<tbody>
<tr>
<td>This set of standards requires students:</td>
<td></td>
</tr>
<tr>
<td>- to write expressions in equivalent forms to reveal key quantities in terms of its context.</td>
<td></td>
</tr>
<tr>
<td>- to choose and use appropriate mathematics to analyze situations.</td>
<td></td>
</tr>
<tr>
<td>For this part of the standards, students recognize that the linear expression mx+b has two terms, m is a coefficient, and b is a constant.</td>
<td></td>
</tr>
<tr>
<td>Students should recognize that in the expression 2x+1, “2” is the coefficient, “2” and “x” are factors, and “1” is a constant, as well as “2x” and “1” being terms of the binomial expression. Development and proper use of mathematical language is an important building block for future content. Using real-world context examples, the nature of algebraic expressions can be explored.</td>
<td></td>
</tr>
<tr>
<td><strong>Example:</strong> The height (in feet) of a balloon filled with helium can be expressed by 5+6.3s where s is the number of seconds since the balloon was released. Identify and interpret the terms and coefficients of the expression.</td>
<td></td>
</tr>
</tbody>
</table>

*September 2017 Update - Feedback*

NC Math 1 Unit 2 Linear Functions

Return to: Standards Addressed in this Unit
Students extend beyond simplifying an expression and address interpretation of the components in an algebraic expression.

NC.M1.A-SSE.1b
*Interpret the structure of expressions.*
Interpret expressions that represent a quantity in terms of its context.

b. Interpret a linear, exponential, or quadratic expression made of multiple parts as a combination of entities to give meaning to an expression.

---

**Concepts and Skills**

<table>
<thead>
<tr>
<th>Pre-requisite</th>
</tr>
</thead>
<tbody>
<tr>
<td>● Interpret a sum, difference, product, and quotient as a both a whole and as a composition of parts (6.EE.2b)</td>
</tr>
<tr>
<td>● Understand that rewriting expressions into equivalent forms can reveal other relationships between quantities (7.EE.2)</td>
</tr>
<tr>
<td>● Interpret numerical expressions written in scientific notation (8.EE.4)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Connections</th>
</tr>
</thead>
<tbody>
<tr>
<td>● Factor to reveal the zeros of functions and solutions to quadratic equations (NC.M1.A.SSE.3)</td>
</tr>
<tr>
<td>● Creating one and two variable equations (NC.M1.A.CED.1, NC.M1.A.CED.2, NC.M1.A.CED.3)</td>
</tr>
<tr>
<td>● Interpreting changes in the parameters of a linear and exponential function in context (NC.M1.F.LE.5)</td>
</tr>
</tbody>
</table>

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**The Standards for Mathematical Practices**

**Implementing the Standards for Mathematical Practice**

<table>
<thead>
<tr>
<th>Connections</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.</td>
</tr>
<tr>
<td>2 – Reason abstractly and quantitatively.</td>
</tr>
<tr>
<td>4 – Model with mathematics</td>
</tr>
<tr>
<td>7 – Look for and make use of structure.</td>
</tr>
</tbody>
</table>

**Disciplinary Literacy**

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary: exponential expression, quadratic expression

---

**Mastering the Standard in this Unit**

<table>
<thead>
<tr>
<th>Comprehending the Standard</th>
<th>Assessing for Understanding</th>
</tr>
</thead>
<tbody>
<tr>
<td>The set of A-SSE standards requires students:</td>
<td></td>
</tr>
<tr>
<td>● to write expressions in equivalent forms to reveal key quantities in terms of its context.</td>
<td></td>
</tr>
<tr>
<td>Students should understand that working with unsimplified expressions often reveals key information from a context.</td>
<td></td>
</tr>
<tr>
<td><strong>Example:</strong> The expression $20(4x)+500$ represents the cost in dollars of the materials and labor needed to build a square fence with side length $x$ feet around a playground. Interpret the constants and coefficients of the expression in context.</td>
<td></td>
</tr>
</tbody>
</table>
• to choose and use appropriate mathematics to analyze situations.

Students identify parts of an expression as a single quantity and interpret the parts in terms of their context.

**Example:** Given that income from a concert is the price of a ticket times each person in attendance, consider the equation \( I = 4000p - 250p^2 \) that represents income from a concert where \( p \) is the price per ticket. What expression could represent the number of people in attendance?

**Solution:** The equivalent factored form, \( p(4000-250p) \), shows that the income can be interpreted as the price times the number of people in attendance based on the price charged. Students recognize \((4000-250p)\) as a single quantity for the number of people in attendance.

---

**NC.M1.A-CED.1**

*Create equations that describe numbers or relationships.*

Create equations and inequalities in one variable that represent linear, exponential, and quadratic relationships and use them to solve problems.

---

**The Standards for Mathematical Practices**

**Implementing the Standards for Mathematical Practice**

**Connections**

*Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.*

1. – Model with mathematics

**Disciplinary Literacy**

*As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.*

Students should be able to describe the origins of created equations and inequalities and demonstrate its relation to the context.

New Vocabulary: exponential function, quadratic function

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**Mastering the Standard in this Unit**

**Comprehending the Standard**

Students create equations and inequalities in one-variable and use them to solve the problems. In Math I, focus on linear contextual situations

**Assessing for Understanding**

Students should be able to create an equation from a function and use the equation to solve problems.

**Example:** A government buys \( x \) fighter planes at \( z \) dollars each, and \( y \) tons of wheat at \( w \) dollars each. It spends a total of \( B \) dollars, where \( B = xz + yw \). In (a)–(c), write an equation whose solution is the given quantity.
that students can use to create equations and inequalities in one variable and use them to solve problems. It is also important to note that equations can also be created from an associated function. After the students have created an equation, they can use other representations to solve problems, such as graphs and tables. Students in Math I are not responsible for interval notation as a solution. They are to write answers to these inequalities using inequality notation.

| a) The number of tons of wheat the government can afford to buy if it spends a total of $100 million, wheat costs $300 per ton, and it must buy 5 fighter planes at $15 million each.
| b) The price of fighter planes if the government bought 3 of them, in addition to 10,000 tons of wheat at $500 a ton, for a total of $50 million.
| c) The price of a ton of wheat, given that a fighter plane costs 100,000 times as much as a ton of wheat, and that the government bought 20 fighter planes and 15,000 tons of wheat for a total cost of $90 million. |

Students should be able to create equations from various representations, such as verbal descriptions, and use them to solve problems. Example: Fishing Adventures rents small fishing boats to tourists for day-long fishing trips. Each boat can only carry 1200 pounds of people and gear for safety reasons. Assume the average weight of a person is 150 pounds. Each group will require 200 lbs of gear for the boat plus 10 lbs of gear for each person.

- a) Create an inequality describing the restrictions on the number of people possible in a rented boat. Graph the solution set.
- b) Several groups of people wish to rent a boat. Group 1 has 4 people. Group 2 has 5 people. Group 3 has 8 people. Which of the groups, if any, can safely rent a boat? What is the maximum number of people that may rent a boat?

NC.M1.A.REI.12
Represent and solve equations and inequalities graphically
Represent the solutions of a linear inequality or a system of linear inequalities graphically as a region of the plane.

Pre-requisite
- Solve two-step linear inequalities (7.EE.4b)
- Solve linear inequalities in one variable (NC.M1.A.REI.3)
- Understand every point on a graph is a solution to its associated equation (NC.M1.A.REI.10)

Connections
- Create one variable linear inequalities and use the inequality to solve problems (NC.M1.A-CED.1)
- Create a system of linear inequalities to model a situation in context (NC.M1.A-CED.3)

The Standards for Mathematical Practices
Implementing the Standards for Mathematical Practice

Connections
- Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.
  - 5 – Use appropriate tools strategically
  - 6 – Attend to precision

Disciplinary Literacy
As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.
Students should be able to explain the reasoning behind their graphical representation of an inequality or system of inequalities.
### Mastering the Standard

<table>
<thead>
<tr>
<th>Comprehending the Standard</th>
<th>Assessing for Understanding</th>
</tr>
</thead>
</table>
| Students should understand that since there is no way to list every solution to a linear inequality in two variables, the solutions must be represented graphically. Similarly, we recognize linear inequalities to have infinitely many solutions. | Students should be able to represent solutions to linear inequalities as a region of a plane. 

**Example:** What scenario could be modeled by the graph below? (multiple choice)

A. The number of pounds of apples, \( y \), minus two times the number of pounds of oranges, \( x \), is at most 5.
B. The number of pounds of apples, \( y \), minus half the number of pounds of oranges, \( x \), is at most 5.
C. The number of pounds of apples, \( y \), plus two times the number of pounds of oranges, \( x \), is at most 5.
D. The number of pounds of apples, \( y \), plus half the number of pounds of oranges, \( x \), is at most 5.

(NCDPI Math 1 released EOC #2)

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**NC.M1.A-CED.2**

*Create equations that describe numbers or relationships.*

Create and graph equations in two variables to represent linear, exponential, and quadratic relationships between quantities.

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### Concepts and Skills

<table>
<thead>
<tr>
<th>Pre-requisite</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Construct a linear function that models the relationship between two quantities (8.F.4)</td>
</tr>
<tr>
<td>• Graph linear equations (8.EE.6)</td>
</tr>
<tr>
<td>• The graph of a function is the set of ordered pairs consisting of input and a corresponding output (8.F.1)</td>
</tr>
</tbody>
</table>

### The Standards for Mathematical Practices

**Implementing the Standards for Mathematical Practice**

<table>
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<th>Connections</th>
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<td>Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.</td>
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<td>4 – Model with mathematics</td>
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- Understand that the graph of a two variable equation represents the set of all solutions to the equation (NC.M1.A-REI.10)

<table>
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<th>Disciplinary Literacy</th>
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<td>● Interpret parts of an expression in context (NC.M1.A-SSE.1a,b)</td>
<td>As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.</td>
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<tr>
<td>● Creating linear equations for a system (NC.M1.A-CED.3)</td>
<td>Students should be able to describe the origins of created equations and demonstrate its relation to the context.</td>
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<tr>
<td>● Solving for a variable of interest in a formula (NC.M1.A-CED.4)</td>
<td>New Vocabulary: exponential function, quadratic function</td>
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<td>● The graph a function $f$ is the graph of the equation $y = f(x)$ (NC.M1.F-IF.1)</td>
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<td>● Interpret a function’s domain and range in context (NC.M1.F-IF.5)</td>
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<tr>
<td>● Identify key features of linear, exponential and quadratic functions</td>
<td></td>
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<tr>
<td>(NC.M1.F-IF.7)</td>
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<tr>
<td>● Building a function through patterns or by combining other functions</td>
<td></td>
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<tr>
<td>(NC.M1.F-BF.1a, NC.M1.F-BF.1b)</td>
<td></td>
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### Mastering the Standard in this Unit

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| Students create equations in two variables. Graph equations on coordinate axes with labels and scales clearly labeling the axes defining what the values on the axes represent and the unit of measure. Students also select intervals for the scale that are appropriate for the context and display adequate information about the relationship. Students interpret the context and choose appropriate minimum and maximum values for a graph. In Math I, focus on linear contextual situations for students to create equations in two variables. | Students should be able to create two variable equations from various representations, such as verbal descriptions, and use them to solve problems. Students should be able to create a two variable equations, graph the relationship, and use graph to recognize key feature of the graph. **Example:** The FFA had a fundraiser by selling hot dogs for $1.50 and drinks for $2.00. Their total sales were $400.  
   a) Write an equation to calculate the total of $400 based on the hot dog and drink sales.  
   b) Graph the relationship between hot dog sales and drink sales.  
      **Note:** This make a good connection to NC.M1.F-IF.5 |
NC.M1.F-BF.1a

*Build a function that models a relationship between two quantities.*

Write a function that describes a relationship between two quantities.

a. Build linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two ordered pairs (include reading these from a table).

### Pre-requisite

- Construct a function to model a linear relationship (8.F.4)
- Formally define a function (NC.M1.F-IF.1)
- Recognize arithmetic and geometric sequences as linear and exponential functions (NC.M1.F-IF.3)
- Identify situations that can be modeled with linear and exponential functions (NC.M1.F-LE.1)

### Connections

- Create and graph two variable equations (NC.M1.A-CED.2)
- Identify key feature of graphs and tables of functions (NC.M1.F-IF.4)
- Identify and interpret key features of functions from different representations (NC.M1.F-IF.7)
- Translate between explicit and recursive forms (NC.M1.F-BF.2)

### Concepts and Skills

#### Implementing the Standards for Mathematical Practice

**Connections**

*Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.*

4 – Model with mathematics

**Disciplinary Literacy**

*As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.*

Students should be able to justify claims that a sequence defines a linear or exponential relationship.

New Vocabulary: arithmetic sequence, geometric sequence, exponential function

### Mastering the Standard in this Unit

#### Comprehending the Standard

This standard is about building a function from different representations. In this part of the standard, the different representations include: sequences, graphs, verbal descriptions, tables, and ordered pairs.

#### Assessing for Understanding

Students should write functions from verbal descriptions as well as a table of values

**Example:** Suppose a single bacterium lands on one of your teeth and starts reproducing by a factor of 2 every hour. If nothing is done to stop the growth of the bacteria, write a function for the number of bacteria as a function of the number of days.

**Example:** The table below shows the cost of a pizza based on the number of toppings.
This standard pairs well with Interpreting Functions standards, in that the purpose behind building a function is to then use that function to solve a problem.

These functions can be written in function notation (linear or exponential) or as a sequence in explicit or recursive form. Students should recognize explicit form of an arithmetic sequence as an equivalent structure to slope-intercept form of a linear function. Using the concepts of rate of change, students should recognize that the forms of these sequences are one iteration forward from the y-intercept, which gives meaning to the \( n-1 \) notation.

<table>
<thead>
<tr>
<th>Number of Toppings ((n))</th>
<th>Cost ((\mathcal{C}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$12.00</td>
</tr>
<tr>
<td>2</td>
<td>$13.50</td>
</tr>
<tr>
<td>3</td>
<td>$15.00</td>
</tr>
<tr>
<td>4</td>
<td>$16.50</td>
</tr>
</tbody>
</table>

Which function represents the cost of a pizza with \( n \) toppings?

A. \( \mathcal{C}(n)=12+1.5(n-1) \)
B. \( \mathcal{C}(n)=1.5n+12 \)
C. \( \mathcal{C}(n)=12+n \)
D. \( \mathcal{C}(n)=12n \)

*(NCDPI Math 1 released EOC #39)*

**Example:** The height of a stack of cups is a function of the number of cups in the stack. If a 7.5” cup with a 1.5” lip is stacked vertically, determine a function that would provide you with the height based on any number of cups.

**Hint:** Start with height of one cup and create a table, list, graph or description that describes the pattern of the stack as an additional cup is added.

**Example:** There were originally 4 trees in an orchard. Each year the owner planted the same number of trees. In the 29th year, there were 178 trees in the orchard. Which function, \( t(n) \), can be used to determine the number of trees in the orchard in any year, \( n \)?

A. \( t(n)=17829n+4 \)
B. \( t(n)=17829n-4 \)
C. \( t(n)=6n+4 \)
D. \( t(n)=29n-4 \)

*(NCDPI Math 1 released EOC #42)*

Students should write linear relationships as a sequence in explicit or recursive form.

**Example:** Investigate the following sequence:
a) Create a recursive formula for the number of stars in the next pattern.
b) Build an explicit formula for the number of stars in the nth pattern.
c) How many stars are in the 43rd pattern?

(www.visualpatterns.org)

NC.M1.A-REI.10
Represent and solve equations and inequalities graphically
Understand that the graph of a two variable equation represents the set of all solutions to the equation.

Pre-requisite
- Use substitution to determine if a number if a solution (6.EE.5)
- Graphing lines (8.EE.5, 8.EE.6, 8.F.3)
- Analyze and solve pairs of simultaneous linear equations by graphing and substitution (8.EE.8)
- Understanding functions as a rule that assigns each input with exactly one output (8.F.1)

Connections
- Creating and graphing two-variable equations (NC.M1.A-CED.2)
- Understanding that the relationship between the solution of system of equations and the associated equation (NC.M1.A-REI.11)
- Representing the solutions to linear inequalities (NC.M1.A-REI.12)
- Relating a function to its graph, domain and range of a function (NC.M1.F-IF.1, NC.M1.F-IF.2, NC.M1.F-IF.5)

The Standards for Mathematical Practices
Implementing the Standards for Mathematical Practice

Connections
Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

Disciplinary Literacy
As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to discuss the solutions to a two variable equation and the link to a function.
Students understand that the graph of an equation is the set of all ordered pairs that make that equation a true statement.

This standard contains no limitation and so applies to all function types, including those functions that a student cannot yet algebraically manipulate.

Students can explain and verify that every point \((x, y)\) on the graph of an equation represents all values for \(x\) and \(y\) that make the equation true.

Example: Consider three points in the plane, \(P=(-4,0)\), \(Q=(-1,12)\) and \(R=(4,32)\).

a) Find the equation of the line through \(P\) and \(Q\).

b) Use your equation in (a) to show that \(R\) is on the same line as \(P\) and \(Q\).

(https://www.illustrativemathematics.org/content-standards/HSA/REI/D/10/tasks/1066)

Example: Which of the following points are on the graph of the equation \(-5x+2y=20\)? Which of the following points are of the graph of the equation? How do you know?

A. \((4, 0)\)
B. \((0, 10)\)
C. \((-1, 7.5)\)
D. \((2.3, 5)\)

NC.M1.A-REI.11

Represent and solve equations and inequalities graphically

Build an understanding of why the \(x\)-coordinates of the points where the graphs of two linear, exponential, or quadratic equations \(y = f(x)\) and \(y = g(x)\) intersect are the solutions of the equation \(f(x) = g(x)\) and approximate solutions using a graphing technology or successive approximations with a table of values.

Concepts and Skills

Pre-requisite
- Solving multi-step linear equations (8.EE.7)
- Analyze and solve pairs of simultaneous linear equations by graphing and substitution (8.EE.8)
- Understand every point on a graph is a solution to its associated equation (NC.M1.A-REI.10)

Connections
- Creating and solving one variable equations and systems of equations (NC.M1.A-CED.1, NC.M1.A-CED.3)
- Solving systems of equations (NC.M1.A-REI.6)

The Standards for Mathematical Practices

Implementing the Standards for Mathematical Practice

Connections
Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics
6 – Attend to precision

Disciplinary Literacy
As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary: exponential function, quadratic function
Comprehending the Standard

For a complete understanding, students will need exposure to both parts of this standard.

First, students should be able to see the connection between graphs and tables of two functions, the points they have in common and the truthfulness of the equation. Because \( f(x) = g(x) \) when \( x = 3 \), 3 is the solution to the equation \( 2x - 4 = 12x + 12 \). (As an extension, students could write an inequality to describe the relationship between the functions when \( x < 3 \) and when \( x > 3 \).)

In Math 1, students are expected to solve linear systems of equations algebraically. All other systems should be solved with technology, tables, and graphs.

Second, students should be able to use a system of equations to solve systems of equations.

\[
\begin{array}{c|c}
 x & f(x) = 2x - 4 \\
\hline
 0 & -4 \\
 1 & -2 \\
 2 & 0 \\
 3 & 2 \\
 4 & 4 \\
\end{array}
\]

\[
\begin{array}{c|c}
 x & g(x) = \frac{1}{2}x + \frac{1}{2} \\
\hline
 0 & .5 \\
 1 & 1 \\
 2 & 1.5 \\
 3 & 2 \\
 4 & 2.5 \\
\end{array}
\]

For example:

Solve: \( 3x^2 - 2x + 1 = \frac{1}{2}x + 5 \)

Rewrite the equations as a system of equations: \( f(x) = 3x^2 - 2x + 1 \) \( g(x) = \frac{1}{2}x + 5 \)

Using technology, graph the equations and look for points of intersection, where the same \( x \) produces \( f(x) = g(x) \).

In Math 1, students are expected to solve linear equations using inverse operations and quadratic equations with square roots and factoring. In all other equations, such as exponential equations, solutions should be approximated with technology, tables and graphs.

Assessing for Understanding

Students should be able to find approximate solutions to linear equations using technology, tables and graphs.

**Example:** The functions \( f(m) = 18 + 0.4m \) and \( g(m) = 11.2 + 0.54m \) give the lengths of two different springs in centimeters, as mass is added in grams, \( m \), to each separately.

a) Graph each equation on the same set of axes.
b) What mass makes the springs the same length?
c) What is the length at that mass?
d) Write a sentence comparing the two springs.
NC.M1.G-GPE.5

*Use coordinates to prove simple geometric theorems algebraically.*

Use coordinates to prove the slope criteria for parallel and perpendicular lines and use them to solve problems.

- Determine if two lines are parallel, perpendicular, or neither.
- Find the equation of a line parallel or perpendicular to a given line that passes through a given point.

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### Concepts and Skills

#### Pre-requisite

- Calculating rate of change given two points, a table or a graph (8.F.4)
- Derive the equation for a line in the coordinate plane (8.EE.6)

#### Connections

- Calculating and interpreting rate of change for a function (NC.M1.F-IF.6)
- Using coordinates to solve geometric problems algebraically (NC.M1.G-GPE.4)
- Analyze functions using different representations (NC.M1.F-IF.7, NC.M1.F-IF.9)
- Using concepts of points lines and planes to develop definitions of rigid motions in the plane (NC.M2.G-CO.2, NC.M2.G-CO.3, NC.M2.G-CO.4)
- Prove theorems about lines (NC.M2.G-CO.9)

### The Standards for Mathematical Practices

#### Implementing the Standards for Mathematical Practice

- **Connections**
  
  *Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.*
  
  3 – Construct viable arguments and critique the reasoning of others.
  
  8 – Look for and express regularity in repeated reasoning.
  
  - The slope formula is a generalization where students notice general methods and/or shortcuts for performing mathematical calculations.

#### Disciplinary Literacy

**As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication. The following vocabulary is new to this course and supported by this standard:**

- Compare and contrast the equations of parallel and perpendicular lines.
- What similarities/differences must be present for parallel lines? Perpendicular lines? Intersecting lines?

### Mastering the Standard in this Unit

#### Comprehending the Standard

Students in 8th grade determine the slope and write the equation of non-vertical lines given two points, a table or graph. This standard is an extension and an application of this work as it asks students to compare two or more lines based on the characteristics of the lines presented.

#### Assessing for Understanding

Given coordinates, students can compare the characteristics, slopes and intercepts, of two or more lines. Student should
- **Parallelism** – same slope
  \[ m_1 = m_2, \text{ where } m = \frac{\Delta y}{\Delta x} \]
- **Perpendicularity** – slopes are opposite reciprocals OR slopes have a product of \(-1\).
  \[ m_1 m_2 = -1, \text{ where } m = \frac{a}{b} \]
- **Intersecting** – have completely different rates of change. It is useful to note that perpendicular lines are a subset of intersecting lines on coordinate plane.
  \[ m_1 m_2 = -1, \text{ where } m = \frac{\Delta y}{\Delta x} \]

The slope formula is an appropriate generalization and should be developed through SMP 8 where students notice general methods and/or shortcuts for performing mathematical calculations. This is based on what students know about rate of change (slope) from MS mathematics.

Students should be able to determine if two lines are parallel, perpendicular or intersecting based on the slopes of the two lines.

**Example:** Investigate the slopes of each of the sides of the rectangle ABCD (pictured on the right). What do you notice about the slopes of the sides that meet at a right angle? What do you notice about the slopes of the opposite sides that are parallel? Can you generalize what happens when you multiply the slopes of perpendicular lines?

Students should be able to find the slope and/or endpoint(s) of a line given the graph or coordinates of a line parallel or perpendicular to the given line.

**Example:** Suppose a line \( k \) in a coordinate plane has slope \( \frac{c}{d} \).

a) What is the slope of a line parallel to \( k \)? Why must this be the case?

b) What is the slope of a line perpendicular to \( k \)? Why does this seem reasonable?

Students should be able to write the equation of line parallel or perpendicular to a given line.

**Example:** Two points \( A(0, -4), B(2, -1) \) determine line \( AB \).

a) What is the equation of the line \( AB \)?

b) What is the equation of the line perpendicular to line \( AB \), passing through the point \( (2, -1) \)?
NC.M1.F-IF.3
Understand the concept of a function and use function notation.
Recognize that recursively and explicitly defined sequences are functions whose domain is a subset of the integers, the terms of an arithmetic sequence are a subset of the range of a linear function, and the terms of a geometric sequence are a subset of the range of an exponential function.

Pre-requisite
- Interpret the equation $y = mx + b$ as being from a linear function and compare to nonlinear functions (8.F.3)
- Define a function and use functions notation (NC.M1.F-IF.1)
- Evaluating functions (NC.M1.F-IF.2)

Connections
- Relating the domain and range to a context (NC.M1.F-IF.5)
- Analyzing linear and exponential functions (NC.M1.F-IF.7)
- Build linear and exponential functions (NC.M1.F-BF.1)
- Translate between explicit and recursive forms (NC.M1.F-BF.2)
- Identify situations that can be modeled with linear and exponential functions (NC.M1.F-LE.1)

Concepts and Skills

The Standards for Mathematical Practices
Implementing the Standards for Mathematical Practice

Connections
Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

Disciplinary Literacy
As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to explain a function written in recursive form using subset notation.
New Vocabulary: arithmetic sequence, geometric sequence, explicit form, recursive form, exponential function

Mastering the Standard in this Unit

Assessing for Understanding

Example: A theater has 60 seats in the first row, 68 seats in the second row, 76 seats in the third row, and so on in the same increasing pattern.

- a) If the theater has 20 rows of seats, how many seats are in the twentieth row?
- b) Explain why the sequence is considered a function.
- c) What is the domain of the sequence? Explain what the domain represents in context.
- d) What is the range of the sequence? Explain what the range represents in context.
NC.M1.F-BF.2

*Build a function that models a relationship between two quantities.*

Translate between explicit and recursive forms of arithmetic and geometric sequences and use both to model situations.

### Concepts and Skills

#### Pre-requisite
- Construct a function to model a linear relationship (8.F.4)
- Formally define a function (NC.M1.F-IF.1)
- Recognize sequences as function and link arithmetic sequences to linear functions and geometric sequences to exponential functions (NC.M1.F-IF.3)
- Build functions from arithmetic and geometric sequences (NC.M1.F-BF.1a)

#### Connections

**The Standards for Mathematical Practices**

**Implementing the Standards for Mathematical Practice**

**Connections**

*Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.*

4 – Model with mathematics

**Disciplinary Literacy**

*As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.*

Students should be able to explain their model in context.

New Vocabulary: arithmetic sequence, geometric sequence, explicit form, recursive form

### Mastering the Standard in this Unit

#### Comprehending the Standard

Students should be able to use both the explicit and recursive forms of arithmetic sequences where the explicit form is a linear function.

Students are expected to use formal notation:
- \(a_n\) (NOW)
- \(a_{n-1}\) (PREVIOUS)
- \(a_{n+1}\) (NEXT)

(Students can use NEXT-NOW notation as they learn to recursive functions but will need to move to formal notation.)

#### Assessing for Understanding

Students should be able to build explicit and recursive forms of arithmetic sequences.

**Example:** A concert hall has 58 seats in Row 1, 62 seats in Row 2, 66 seats in Row 3, and so on. The concert hall has 34 rows of seats.

a) Write a recursive formula to find the number of seats in each row. How many seats are in row 5?

b) Write the explicit formula to determine which row has 94 seats?

**Example:** Given the sequence defined by the function \(a_{n+1} = a_n + 12\) with \(a_1 = 4\). Write an explicit function rule.

**Note:** Student may interpret 4 as the y-intercept since it is the first value; however, attending to the notation when \(x = 1, y = 4\). Thus, the y-intercept for the explicit form is -8.
This standard should be tied to NC.M1.F-IF.3, recognizing patterns and linking to function types.

NC.M1.A-REI.1

Understand solving equations as a process of reasoning and explain the reasoning.
Justify a chosen solution method and each step of the solving process for linear and quadratic equations using mathematical reasoning.

Concepts and Skills

Pre-requisite
- Students have been using properties of operations and equality throughout middle school. (6.EE.3, 7.EE.1, 7.EE.4). This is the first time that justification is required by a content standard.
- Solve multi-step equations (8.EE.7)

Connections
- Understand the relationship between factors of a quadratic equation and the solution of the equation (NC.M1.A-APR.3)
- Create and solve one variable linear and quadratic equations (NC.M1.A-CED.1)
- Solve for a quantity of interest in a formula (NC.M1.A-CED.4)

The Standards for Mathematical Practices

Implementing the Standards for Mathematical Practice

Connections
Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.
3 – Construct viable arguments and critique the reasoning of others

Disciplinary Literacy
As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to defend their method of solving an equation and each step of the solving process.
New Vocabulary: quadratic equation

Mastering the Standard in this Unit

Comprehending the Standard
When solving equations, students will use the properties of equality to justify and explain each step obtained from the previous step, assuming the original equation has a solution, and develop an argument that justifies their method.
Properties of operations can be used to change expressions on either side of the equation to

Assessing for Understanding
Students should be able to justify a chosen solution method and justify each step in the process. This would be a good opportunity to discuss efficiency.
Example: Below are two methods to solve the same equation. Justify each step in the solving process. Which method do you prefer? Why?
equivalent expressions.
In the properties of equality, adding the same
term to both sides of an equation or multiplying
both sides by a non-zero constant produces an
equation with the same solutions.

Students do not have to name the property, but
can describe the property using mathematical
reasoning.

*For example:* Transforming $2x - 5 = 7$ to
$2x = 12$ is possible because $5 = 5$, so adding
the same quantity to both sides of an equation
makes the resulting equation true as well. *Knowing this is the Addition Property of
Equality is not the point of this standard.*

<table>
<thead>
<tr>
<th>Method 1:</th>
<th>Method 2:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5(x + 3) - 3x = 55$</td>
<td>$5(x + 3) - 3x = 55$</td>
</tr>
<tr>
<td>$5x + 15 - 3x = 55$</td>
<td>$\frac{5}{5}$</td>
</tr>
<tr>
<td>$2x + 15 = 55$</td>
<td>$\frac{5}{5}$</td>
</tr>
<tr>
<td>$2x + 15 - 15 = 55 - 15$</td>
<td>$\frac{x + 3 - \frac{3}{5}x = 11}{\frac{2}{5}}$</td>
</tr>
<tr>
<td>$2x = 40$</td>
<td>$\frac{2}{5}x + 3 = 11$</td>
</tr>
<tr>
<td>$\frac{2x}{2} = \frac{40}{2}$</td>
<td>$\frac{2}{5}x + 3 - 3 = 11 - 3$</td>
</tr>
<tr>
<td>$x = 20$</td>
<td>$\frac{2}{5}x = 8$</td>
</tr>
</tbody>
</table>

Students should be able to critique the solving process of
others, recognize incorrect steps and provide corrective action
to the process.

**Example:** The following is a student solution to the inequality
$5 - \frac{x-2}{18} \leq \frac{x-4}{6}$.

a) There are two mathematical errors in this work. Identify at what step each
mathematical error occurred and explain why it is mathematically incorrect.
b) How would you help the student understand his mistakes?
c) Solve the inequality correctly.

Note: While this standard does not cover inequalities, this could be a good extension.
NC.M1.F-LE.5
Interpret expressions for functions in terms of the situation they model.
Interpret the parameters $a$ and $b$ in a linear function $f(x) = ax + b$ or an exponential function $g(x) = ab^x$ in terms of a context.

### Concepts and Skills

#### Pre-requisite
- Construct a function to model a linear relationship and interpret rate of change and initial value (8.F.4)
- Compare the coefficients and constants of linear equations in similar form (8.EE.b)
- Identify and interpret parts of expression (NC.M1.A-SSE.1a, NC.M1.A-SSE.1b)

#### Connections
- Identify and interpret key features of functions from different representations (NC.M1.F-IF.7)

### The Standards for Mathematical Practices

#### Implementing the Standards for Mathematical Practice

**Connections**

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

**Disciplinary Literacy**

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary: exponential function

### Mastering the Standard in this Unit

#### Comprehending the Standard

Students should know the meaning of the parameters in linear functions in the context of the situation.

Use real-world situations to help students understand how the parameters of linear functions depend on the context.

In a linear function $y=ax+b$ the value of “$a$” represents the slope (constant rate of change) while “$b$” represents the y intercept (initial value).

#### Assessing for Understanding

Students should be able to describe the effects of changes to the parameters of a linear and exponential functions.

**Example:** A plumber who charges $50 for a house call and $85 per hour can be expressed as the function $y = 85x + 50$.

If the rate were raised to $90 per hour, how would the function change?

Students should be able to interpret the parameters of a linear and exponential function.

**Example:** Lauren keeps records of the distances she travels in a taxi and what it costs:

<table>
<thead>
<tr>
<th>Distance $d$ in miles</th>
<th>Fare $f$ in dollars</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>8.25</td>
</tr>
<tr>
<td>5</td>
<td>12.75</td>
</tr>
<tr>
<td>11</td>
<td>26.25</td>
</tr>
</tbody>
</table>

a) If you graph the ordered pairs $(d, f)$ from the table, they lie on a line. How can this be determined without graphing them?
### NC.M1.F-IF.5

**Interpret functions that arise in applications in terms of the context.**

Interpret a function in terms of the context by relating its domain and range to its graph and, where applicable, to the quantitative relationship it describes.

#### Concepts and Skills

<table>
<thead>
<tr>
<th>Pre-requisite</th>
</tr>
</thead>
<tbody>
<tr>
<td>● In middle school, students only informally considered restrictions to the domain and range based on context, such as understanding that measurements cannot be negative.</td>
</tr>
<tr>
<td>● Interpret parts of expressions in context (NC.M1.A-SSE.1a, NC.M1.A-SSE.1b)</td>
</tr>
<tr>
<td>● Every point on the graph of an equation is a solution to the equation (NC.M1.A-REI.10)</td>
</tr>
<tr>
<td>● Formally define a function (NC.M1.F-IF.1)</td>
</tr>
<tr>
<td>● Evaluating functions and interpret in context (NC.M1.F-IF.2)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Connections</th>
</tr>
</thead>
<tbody>
<tr>
<td>● Recognize the domain of sequences (NC.M1.F-IF.3)</td>
</tr>
<tr>
<td>● Identify key feature of graphs and tables of functions (NC.M1.F-IF.4)</td>
</tr>
<tr>
<td>● Analyze linear, quadratic, and exponential functions to identify key features (NC.M1.F-IF.7)</td>
</tr>
</tbody>
</table>

#### The Standards for Mathematical Practices

**Implementing the Standards for Mathematical Practice**

- **Connections**
  
  *Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.*

  4 – Model with mathematics

- **Disciplinary Literacy**
  
  *As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.*

#### Mastering the Standard in this Unit

<table>
<thead>
<tr>
<th>Comprehending the Standard</th>
<th>Assessing for Understanding</th>
</tr>
</thead>
</table>

September 2017 Update - Feedback

NC Math 1 Unit 2 Linear Functions

Return to: Standards Addressed in this Unit
Students should be able to identify a reasonable domain and range to its graph as well as to a contextual situation. Domain of graphs should be taught in the context of the situation they represent.

Graphs represented should be both discrete and continuous forms. Students do not need to know the terminology discrete and continuous, but they should be able to identify which is appropriate for each contextual situation.

- Students should be able to identify a reasonable domain and range to its graph as well as to a contextual situation.

**Example:** Collin noticed that various combinations of nickels and dimes could add up to $0.65.
- Let $x$ equal the number of nickels.
- Let $y$ equal the number of dimes.

What is the domain where $y$ is a function of $x$ and the total value is $0.65$? (NCDPI Math 1 released EOC #37)

- a) $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13\}$
- b) $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13\}$
- c) $\{0, 1, 3, 5, 7, 9, 11, 13\}$
- d) $\{1, 3, 5, 7, 9, 11, 13\}$

**Example:** Jennifer purchased a cell phone and the plan she decided upon charged her $50 for the phone and $0.10 for each minute she is on the phone. (The wireless carrier rounds up to the half minute.) She has budgeted $100 for her phone bill. What would be the appropriate domain for the cost as a function of the total minutes she used the phone? Describe what the point $(10, 51)$ represents in the problem.

**Example:** Oakland Coliseum, home of the Oakland Raiders, is capable of seating 63,026 fans. For each game, the amount of money that the Raiders’ organization brings in as revenue is a function of the number of people, $n$, in attendance. If each ticket costs $30, find the domain of this function.

At a game, the Raiders has decided to honor fans who served in the military. For this event, the Raiders will be giving away 1,500 tickets to military families. How does this effect the domain and range of the function? What does this mean for the Raiders and their fans?

**Example:** An all-inclusive resort in Los Cabos, Mexico provides everything for their customers during their stay including food, lodging, and transportation. Use the graph at the right to describe the domain of the total cost function.
NC.M1.S-ID.9
*Interpret linear models.*
Distinguish between association and causation.

### Concepts and Skills

**Pre-requisite**
- Construct and interpret scatterplots for two-variable data and describe patterns of association (8.SP.1)
- Fit a regression line to linear data using technology (NC.M1.S-ID.6a)
- Assess linearity by analyzing residuals (NC.M1.S-ID.6b)

**Connections**
- Fit a function to exponential data using technology (NC.M1.S-ID.6c)

### The Standards for Mathematical Practices

**Implementing the Standards for Mathematical Practice**

**Connections**
Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

**Vocabulary**
As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

### Mastering the Standard in this Unit

#### Comprehending the Standard
In working with bivariate data in MS, students have previously investigated patterns of association between two quantities. Specifically, positive and negative associations and linear and non-linear associations.

This standard addresses an often made misconception in regard to association, correlation and causation. Association indicates a relationship between two or more variables and correlation indicates the degree of association between two quantities. Causation, on the other hand, implies a cause and effect relationship when a strong relationship is observed.

#### Assessing for Understanding
Students will recognize that association does not imply causation.

**Example:** The following graph shows the correlation between Letters in Winning Word of Scripps National Spelling Bee and Number of people killed by venomous spiders. How does the graph support the phrase: association does not imply causation?

For more examples, explore the site [http://tylervigen.com/](http://tylervigen.com/).

Students will determine if statements of causation are reasonable or not and justify their opinion.
Determining causation goes beyond the idea of mere association or a high degree of correlation and requires the design and analysis of a randomized experimental process.

**Example:** A study found a strong, positive correlation between the number of cars owned and the length of one’s life. Larry concludes that owning more cars means you will live longer. Does this seem reasonable? Explain your answer.

**Example:** Choose two variables that could be correlated because one is the cause of the other; defend and justify the selection of variables.

**NC.M1.F-IF.9**

*Analyze functions using different representations.*

Compare key features of two functions (linear, quadratic, or exponential) each with a different representation (symbolically, graphically, numerically in tables, or by verbal descriptions).

### Concepts and Skills

**Pre-requisite**

- Compare properties of two functions each represented in different ways (8.F.2)
- Formally define a function (NC.M1.F-IF.1)
- Identify key feature of graphs and tables of functions (NC.M1.F-IF.4)
- Identify and interpret key features of functions from different representations (NC.M1.F-IF.7)
- Rewrite quadratic functions to identify key features (NC.M1.F-IF.8a)
- Interpret and explain growth and decay rates for an exponential function (NC.M1.F-IF.8b)

**Connections**

*The Standards for Mathematical Practices*

**Implementing the Standards for Mathematical Practice**

**Connections**

*Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.*

4 – Model with mathematics

**Disciplinary Literacy**

*As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.*

Students should be able to justify their use of a representation to make the comparison.

New Vocabulary: exponential function, quadratic function
### Comprehending the Standard

Students should compare two functions in two different forms. The function types may be the same (linear & linear), but the representations should be different (e.g. numerical & graphical).

<table>
<thead>
<tr>
<th>$x$</th>
<th>$g(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>−7</td>
<td>2</td>
</tr>
<tr>
<td>−5</td>
<td>3</td>
</tr>
<tr>
<td>−3</td>
<td>4</td>
</tr>
<tr>
<td>−1</td>
<td>5</td>
</tr>
</tbody>
</table>

### Assessing for Understanding

**Example:** Dennis compared the y-intercept of the graph of the function $f(x) = 3x + 5$ to the y-intercept of the graph of the linear function that includes the points in the table below.

What is the difference when the y-intercept of $f(x)$ is subtracted from the y-intercept of $g(x)$?

- A) −11.0
- B) −9.3
- C) 0.5
- D) 5.5

(NCDPI Math 1 released EOC #22)

**Example:** Mario compared the slope of the function graphed to the right to the slope of the linear function that has an $x$-intercept of $\frac{4}{3}$ and a $y$-intercept of $−2$.

What is the slope of the function with the smaller slope?

- A) $\frac{1}{5}$
- B) $\frac{1}{3}$
- C) 3
- D) 5

(NCDPI Math 1 EOC released #25)

**NC.M1.F-IF.7**

*Analyze functions using different representations.*

Analyze linear, exponential, and quadratic functions by generating different representations, by hand in simple cases and using technology for more complicated cases, to show key features, including: domain and range; rate of change; intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximums and minimums; and end behavior.

---

**Concepts and Skills**

- Interpret $y = mx + b$ as being linear (8.F.3)
- Determine rate of change and initial value of linear functions from tables and graphs (8.F.4)

---

**The Standards for Mathematical Practices**

Implementing the Standards for Mathematical Practice

- **Connections**
  - Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.
  - 4 – Model with mathematics

---

*September 2017 Update - Feedback*
- Interpret parts of expressions in context (NC.M1.A-SSE.1a, NC.M1.A-SSE.1b)
- Formally define a function (NC.M1.F-IF.1)
- Evaluating functions and interpret in context (NC.M1.F-IF.2)
- Identify key feature of graphs and tables of functions (NC.M1.F-IF.4)

**Connections**

- Creating and graphing two variable equations (NC.M1.A-CED.2)
- Solving systems of equations (NC.M1.A-REI.6)
- Recognize the domain of sequences as integers (NC.M1.F-IF.3)
- Relate domain and range of a function to its graph (NC.M1.F-IF.5)
- Calculate the average rate of change (NC.M1.F-IF.6)
- Use equivalent forms of quadratic and exponential function to reveal key features (NC.M1.F-IF.8a, NC.M1.F-IF.8b)
- Compare key features of two functions in different representations (NC.M1.F-IF.9)
- Build functions that describe a relationship between two quantities (NC.M1.F-BF.1a, NC.M1.F-BF.1b)
- Identify situations that can be modeled with linear and exponential functions (NC.M1.F-LE.1)
- Interpret the parameters of a linear and exponential function in context (NC.M1.F-LE.5)

**Disciplinary Literacy**

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to justify their use of a representation.

New Vocabulary: exponential function, quadratic function

---

**Mastering the Standard in this Unit**

<table>
<thead>
<tr>
<th>Comprehending the Standard</th>
<th>Assessing for Understanding</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students should identify the key features of linear functions.</td>
<td>Students should be able to identify key feature of linear, quadratic and exponential functions from the symbolic representation.</td>
</tr>
<tr>
<td>Students should be aware of the key functions typically associated with each function type. Linear functions – domain &amp; range, rate of change, intercepts, increasing/decreasing</td>
<td><strong>Example:</strong> Describe the key features of the graph ( f(x) = \frac{-2}{3}x + 8 ) and use the key features to create a sketch of the function.</td>
</tr>
<tr>
<td></td>
<td>Students should be able to identify key feature of linear functions from the graphical representation.</td>
</tr>
</tbody>
</table>
NC.M1.S-ID.6a

**Summarize, represent, and interpret data on two categorical and quantitative variables.**

Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

a. Fit a least squares regression line to linear data using technology. Use the fitted function to solve problems.

---

### Concepts and Skills

**Pre-requisite**
- Construct and interpret scatterplots for two-variable data and describe patterns of association (8.SP.1)
- Informally fit a straight line assess the model fit judging the closeness of the data to line (8.SP.2)
- Analyze patterns and describe relationships between variables in context. (NC.M1.S-ID.8)

**Connections**
- Assess linearity by analyzing residuals (NC.M1.S-ID.6b)
- Fit a function to exponential data using technology and use the model to solve problems (NC.M1.S-ID.6c)
- Use technology to analyze patterns and describe relationships between two variables in context. (NC.M1.S-ID.7)
- Distinguish between association and causation (NC.M1.S-ID.9)
- Write a function that describes a relationship between two quantities (NC.M1.F-BF.1)
- Identify situations that can be modeled with linear and exponential functions and justify the appropriate model (NC.M1.F-LE.1)

---

### The Standards for Mathematical Practices

#### Implementing the Standards for Mathematical Practice

**Connections**

*Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.*

**Vocabulary**

*As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication. The following vocabulary is new to this course and supported by this standard:*
In 8th grade, students created scatter plots and described patterns of association between two quantities. They also informally fit a straight line to data based on how closely the data points resembled a line. That knowledge is extended to fitting a linear regression equation to a set of data using technology. Technology includes graphing calculators, computer software/programs and web-based applets and tools.

The initial exploration with technology should include a discussion of domain and range and their relationship to the graphing window. Most technology tools include an automatic feature that graphs data within a window representative of the data, however understanding of the graphing window can lead to further discussions about domain, range, interpolation and extrapolation.

### Mastering the Standard in this Unit

#### Comprehending the Standard

In 8th grade, students created scatter plots and described patterns of association between two quantities. They also informally fit a straight line to data based on how closely the data points resembled a line. That knowledge is extended to fitting a linear regression equation to a set of data using technology. Technology includes graphing calculators, computer software/programs and web-based applets and tools.

The initial exploration with technology should include a discussion of domain and range and their relationship to the graphing window. Most technology tools include an automatic feature that graphs data within a window representative of the data, however understanding of the graphing window can lead to further discussions about domain, range, interpolation and extrapolation.

#### Assessing for Understanding

Students can represent data on a scatter plot using an appropriate scale and describe the relationship between two quantitative variables.

<table>
<thead>
<tr>
<th>HS Population</th>
<th># of active band members</th>
</tr>
</thead>
<tbody>
<tr>
<td>1200</td>
<td>150</td>
</tr>
<tr>
<td>1450</td>
<td>155</td>
</tr>
<tr>
<td>900</td>
<td>100</td>
</tr>
<tr>
<td>1500</td>
<td>125</td>
</tr>
<tr>
<td>1400</td>
<td>125</td>
</tr>
<tr>
<td>1005</td>
<td>120</td>
</tr>
</tbody>
</table>

### Example: Represent the data from the table below in a scatter plot. Determine if and what the relationship is between the population of each high school and the number of active band members.

<table>
<thead>
<tr>
<th>HS Population</th>
<th># of active band members</th>
</tr>
</thead>
<tbody>
<tr>
<td>1200</td>
<td>150</td>
</tr>
<tr>
<td>1450</td>
<td>155</td>
</tr>
<tr>
<td>900</td>
<td>100</td>
</tr>
<tr>
<td>1500</td>
<td>125</td>
</tr>
<tr>
<td>1400</td>
<td>125</td>
</tr>
<tr>
<td>1005</td>
<td>120</td>
</tr>
</tbody>
</table>

### Example: The data gives the number of miles driven and advertised price for 11 used models of a particular car.

<table>
<thead>
<tr>
<th>Miles (thousands)</th>
<th>Price ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>22</td>
<td>17,998</td>
</tr>
<tr>
<td>29</td>
<td>16,450</td>
</tr>
<tr>
<td>35</td>
<td>14,998</td>
</tr>
<tr>
<td>39</td>
<td>13,998</td>
</tr>
<tr>
<td>45</td>
<td>14,599</td>
</tr>
<tr>
<td>49</td>
<td>14,988</td>
</tr>
<tr>
<td>55</td>
<td>13,599</td>
</tr>
<tr>
<td>56</td>
<td>14,599</td>
</tr>
<tr>
<td>69</td>
<td>11,998</td>
</tr>
<tr>
<td>70</td>
<td>14,450</td>
</tr>
<tr>
<td>86</td>
<td>10,998</td>
</tr>
</tbody>
</table>

- **a)** Use a calculator or graphing technology to make a scatter plot of the data.
- **b)** Find the correlation coefficient for the data above. Describe what the correlation coefficient means in regards to the data.
- **c)** Fit a linear function to model the relationship between miles driven and the price of these cars.
- **d)** How do you know that this is the best-fit model?
- **e)** If a used car is driven 98,000 miles, what will the price be (to the nearest dollar)?
- **f)** If the price of the car is $12,540, how many miles could have been driven (to the nearest thousand)?
NC.M1.S-ID.7
Interpret linear models.
Interpret in context the rate of change and the intercept of a linear model. Use the linear model to interpolate and extrapolate predicted values. Assess the validity of a predicted value.

<table>
<thead>
<tr>
<th>Concepts and Skills</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pre-requisite</strong></td>
</tr>
<tr>
<td>- Interpret the slope and y-intercept of a linear model in context (8.SP.3)</td>
</tr>
<tr>
<td><strong>Connections</strong></td>
</tr>
<tr>
<td>- Fit a regression line to linear data using technology (NC.M1.S-ID.6a)</td>
</tr>
<tr>
<td>- Interpret the parameters in linear or exponential functions in terms of a context (NC.M1.F-LE.5)</td>
</tr>
<tr>
<td>- Interpret key features in context to describe functions relating two quantities (NC.M1.F-IF.4)</td>
</tr>
<tr>
<td>- Calculate and interpret the avg. rate of change for a function (NC.M1.F-IF.6)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>The Standards for Mathematical Practices</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Implementing the Standards for Mathematical Practice</strong></td>
</tr>
<tr>
<td><strong>Connections</strong></td>
</tr>
<tr>
<td>Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.</td>
</tr>
<tr>
<td><strong>Vocabulary</strong></td>
</tr>
<tr>
<td>As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication. The following vocabulary is new to this course and supported by this standard:</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mastering the Standard in this Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Comprehending the Standard</strong></td>
</tr>
<tr>
<td>Students have interpreted the slope and y-intercept of a linear model in 8th grade. This standard expands upon this notion to using the model to make predictions.</td>
</tr>
<tr>
<td><strong>Interpolation</strong> is using the function to predict the value of the dependent variable for an independent variable that is in the midst of the data.</td>
</tr>
<tr>
<td><strong>Extrapolation</strong> is using the function to predict the value of the dependent variable for an independent variable that is outside the range of our data.</td>
</tr>
<tr>
<td><strong>Assessing for Understanding</strong></td>
</tr>
<tr>
<td>Students can interpret the meaning of the rate of change and y-intercept in context and can interpolate and/or extrapolate predicted values using the linear model.</td>
</tr>
<tr>
<td><strong>Example:</strong> Data was collected of the weight of a male white laboratory rat for the first 25 weeks after its birth. A scatterplot of the rat’s weight (in grams) and the time since birth (in weeks) indicates a fairly strong, positive linear relationship. The linear regression equation $W = 100 + 40t$ (where $W$ = weight in grams and $t$ = number of weeks since birth) models the data fairly well.</td>
</tr>
<tr>
<td>a) Explain the meaning of the slope of the linear regression equation in context.</td>
</tr>
<tr>
<td>b) Explain the meaning of the y-intercept of the linear regression equation in context.</td>
</tr>
<tr>
<td>c) Based on the linear regression model, what will be the weight of the rat 10 weeks after birth?</td>
</tr>
<tr>
<td>d) Based on the linear regression model, at how many weeks will the rat be 760 grams?</td>
</tr>
</tbody>
</table>
NC.M1.S-ID.8
*Interpret linear models.*
Analyze patterns and describe relationships between two variables in context. Using technology, determine the correlation coefficient of bivariate data and interpret it as a measure of the strength and direction of a linear relationship. Use a scatter plot, correlation coefficient, and a residual plot to determine the appropriateness of using a linear function to model a relationship between two variables.

### Concepts and Skills

**Pre-requisite**
- Construct and interpret scatterplots for two-variable data and describe patterns of association (8.SP.1)
- Fit a regression line to linear data using technology (NC.M1.S-ID.6a)
- Assess linearity by analyzing residuals (NC.M1.S-ID.6b)

**Connections**
- Identify situations that can be modeled with linear and exponential functions, and justify the most appropriate model (NC.M1.F-LE.1)

### The Standards for Mathematical Practices

#### Implementing the Standards for Mathematical Practice

**Connections**
Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

**Vocabulary**
As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication. The following vocabulary is new to this course and supported by this standard:

### Mastering the Standard in this Unit

#### Comprehending the Standard
In working with bivariate data in MS, students have previously

The correlation coefficient, $r$, is a measure of the strength and direction of a linear relationship between two quantities in a set of data.

The magnitude (absolute value) of $r$ indicates how closely the data points fit a linear pattern.

#### Assessing for Understanding

**Students can interpret the correlation coefficient.**

**Example:** The correlation coefficient of a given data set is 0.97. List three specific things this tells you about the data.

**Students recognize the strength of the association of two quantities based on the scatter plot.**

**Example:** Which correlation coefficient best matches each graph? Explain.
If \( r = \pm 1 \), all points fall exactly on a line. The sign of \( r \) indicates the direction of the relationship. The closer \(|r|\) is to 1, the stronger the correlation and the closer \(|r|\) is to zero, the weaker the correlation.

Instructions for TI-83 and TI-84 series calculators:

1: Go to the [catalog]. Click → 2nd then 0.
2: Scroll down to → DiagnosticOn and press enter twice.

When ‘Done’ appears on the screen the diagnostics are on and the calculator should now calculate the correlation coefficient (\( r \)) automatically when linear regression is performed.

A. \( r = \) _______
B. \( r = \) _______
C. \( r = \) _______

\[
\begin{array}{c|c|c|c|c|c|c|c}
& \text{A.} & \text{B.} & \text{C.} \\
& r = -.48 & r = .98 & r = .88 & r = -.17 & r = 1 & r = .31 & r = -1 \\
\end{array}
\]

Students will be able to analyze patterns in context between two variables and use graphing technology to determine whether a linear model is appropriate for the data.

Example: The following data set indicates the average weekly temperature and the number of sno-cones sold by Sno-Show Sno-cones each week in May for the temperatures noted.

<table>
<thead>
<tr>
<th>Average weekly temperature</th>
<th># of Sno-cones sold</th>
</tr>
</thead>
<tbody>
<tr>
<td>68</td>
<td>500</td>
</tr>
<tr>
<td>74</td>
<td>600</td>
</tr>
<tr>
<td>74</td>
<td>700</td>
</tr>
<tr>
<td>80</td>
<td>800</td>
</tr>
<tr>
<td>82</td>
<td>1200</td>
</tr>
</tbody>
</table>

a) Using technology, sketch a scatter plot of the data.
b) Determine a linear regression model that could represent the data shown.
c) Determine the correlation coefficient.
d) Determine the strength and direction of the linear relationship.
e) Create a residual plot. Is a linear model appropriate for the data shown? Explain.

NOTE: Remind students to turn the Diagnostics on in the graphing calculator so that the correlation coefficient (\( r \)) appears when the regression equation is calculated.
NC.M1.G-GPE.6

*Use coordinates to prove simple geometric theorems algebraically.*

Use coordinates to find the midpoint or endpoint of a line segment.

### Concepts and Skills

**Pre-requisite**
- Finding the distance between points in the coordinate plane (8.G.8)
- (7.RP.2d)

**Connections**
- Use coordinates to solve geometric problems involving polygons (NC.M1.G-GPE.4)
- Prove theorems about lines (NC.M2.G-CO.9)

### The Standards for Mathematical Practices

#### Implementing the Standards for Mathematical Practice

**Connections**

*Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.*

3 – Construct viable arguments and critique the reasoning of others.

8 – Look for and express regularity in repeated reasoning.

- The **midpoint formula** is a generalization where students notice general methods and/or shortcuts for performing mathematical calculations.

**Vocabulary**

*As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication. The following vocabulary is new to this course and supported by this standard:*

### Mastering the Standard in this Unit

#### Comprehending the Standard

The midpoint partitions the ratio of two distinct points on the same line segment into 1:1; thus from either direction the point is the same.

The midpoint is always halfway between the two endpoints. The $x$-coordinate of the midpoint will be the mean of the $x$-coordinates of the endpoints and the $y$-coordinate will be the mean of the $y$-coordinates of the endpoints as indicated through the use of the midpoint formula.

**Example:** Jennifer and Jane are best friends. They placed a map of their town on a coordinate grid and found the point at which each of their house lies. If Jennifer’s house lies at (9, 7) and Jane’s house is at (15, 9) and they wanted to meet in the middle, what are the coordinates of the place they should meet?

**Assessing for Understanding**

Given two points on a line, students can find the point that divides the segment into an equal number of parts.

Given the midpoint and an endpoint, students can use what they know about the midpoint to locate the place they should meet.

*September 2017 Update - Feedback*
This should derived from what students understand about distance. The midpoint formula \((\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})\) is an appropriate generalization and should be developed through SMP 8 where students notice general methods and/or shortcuts for performing mathematical calculations.

### Example:
If you are given the midpoint of a segment and one endpoint, find the other endpoint.

\begin{itemize}
  \item a) Midpoint: (6, 2) endpoint: (1, 3)
  \item b) Midpoint: (-1, -2) endpoint: (3.5, -7)
\end{itemize}

---

**NC.M1.G-GPE.4**

*Use coordinates to prove simple geometric theorems algebraically.*

Use coordinates to solve geometric problems involving polygons algebraically

- Use coordinates to compute perimeters of polygons and areas of triangles and rectangles.
- Use coordinates to verify algebraically that a given set of points produces a particular type of triangle or quadrilateral.

---

### Concepts and Skills

<table>
<thead>
<tr>
<th>Pre-requisite</th>
</tr>
</thead>
<tbody>
<tr>
<td>Finding the distance between points in the coordinate plane (8.G.8)</td>
</tr>
<tr>
<td>Calculating rate of change from two points (8.F.4)</td>
</tr>
<tr>
<td>Using slope to determine parallelism and perpendicularity (NC.M1.G-GPE.5)</td>
</tr>
<tr>
<td>Finding midpoint/endpoint of a line segment, given either (NC.M1.G-GPE.6)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Connections</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometric transformations as functions (NC.M2.F-IF.1)</td>
</tr>
</tbody>
</table>

### The Standards for Mathematical Practices

**Implementing the Standards for Mathematical Practice**

**Connections**

*Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.*

3 – Construct viable arguments and critique the reasoning of others.
- Students must use algebraic reasoning as they solve geometric problems.

8 – Look for and express regularity in repeated reasoning
- The distance formula is a generalization where students notice general methods and/or shortcuts for performing mathematical calculations.

**Disciplinary Literacy**

*As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.*

Students should be able to justify their claim that a set of points forms a particular shape using mathematical reasoning.
## Mastering the Standard in this Unit

### Comprehending the Standard

In upper elementary and middle grades, students calculated the area of triangles and special quadrilaterals using all four quadrants of the coordinate plane. Students also applied geometric measurement to real-world and mathematical problems and made use of properties of two-dimensional figures in order to calculate or estimate their lengths and areas.

This standard emphasizes the use of coordinates to solve geometric problems algebraically and continues with geometric measurement. Students will begin to demonstrate and analyze properties of geometric shapes using equations and graphs. This includes:

- Using previously learned formulas to find the perimeter of polygons and the area of triangles and rectangles.

- Applying the slope to determine right angles in triangles and rectangles (perpendicular lines), to verify parallel sides in geometric figures; and to determine intersecting lines.

- Finding the perimeter of figures by computing the distance between points on the coordinate plane.

The distance formula \( d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \) is an appropriate generalization and should be developed through SMP 8 where students notice general methods and/or shortcuts for performing mathematical calculations. This is based on what students know about finding the length of line segments in the coordinate plane (Pythagorean Theorem) from MS mathematics.

### Assessing for Understanding

Given coordinates of a polygon in the coordinate plane, students should be able to compute the lengths of segments and side lengths of polygons by finding the distance between points in the coordinate plane to:

- calculate the perimeter of polygons

- calculate the area of triangles and rectangles

**Example:** Find the perimeter and area of a polygon with vertices at \( C (-1, 1) \), \( D(3, 4) \), \( E(6, 0) \), \( F(2, -3) \) and \( G(-4, -4) \). Round your answer to the nearest hundredth.

Given coordinates of a polygon in the coordinate plane, students should be able to verify the properties of any triangle or quadrilateral using the slopes of lines and lengths of segments that comprise the figure.

**Example:** Given \( ABC \) with altitude \( CD \), given \( A (-4, -2) \), \( B(8, 7) \), \( C(1, 8) \) and \( D(4, 4) \).

- a) Calculate the area of \( ABC \).
- b) The altitude of a triangle is defined as a line that extends from one vertex of a triangle perpendicular to the opposite side. Verify that \( CD \) is an altitude of \( ABC \).

**Example:** The coordinates for the vertices of quadrilateral \( MNPQ \) are \( M(3, 0) \), \( N(1, 3) \), \( P(-2, 1) \), and \( Q(0, -2) \).

- a) Classify quadrilateral \( MNPQ \).
- b) Identify the properties used to determine your classification.

Given the properties of a rectangle or triangle, students can determine the missing coordinate(s).

**Example:** If quadrilateral \( ABCD \) is a rectangle, where \( A(1, 2) \), \( B(6, 0) \), \( C(10, 10) \) and \( D(x, y) \) is unknown.

- a) Find the coordinates of the fourth vertex Point D.
- b) Verify that \( ABCD \) is a rectangle providing evidence related to the sides and angles.
NC.M1.S-ID.6b

**Summarize, represent, and interpret data on two categorical and quantitative variables.**

Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

b. Assess the fit of a linear function by analyzing residuals.

### Concepts and Skills

**Pre-requisite**
- Fit a regression line to linear data using technology (NC.M1.S-ID.6a)

**Connections**
- Use technology to analyze patterns and describe relationships between two variables in context. (NC.M1.S-ID.7)
- Analyze patterns and describe relationships between variables in context. (NC.M1.S-ID.8)

### The Standards for Mathematical Practices

**Implementing the Standards for Mathematical Practice**

**Connections**

*Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.*

3 – Construct a viable argument and critique the reasoning of others

**Vocabulary**

*As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication. The following vocabulary is new to this course and supported by this standard:*

New Vocabulary: residual

### Mastering the Standard in this Unit

**Comprehending the Standard**

A residual, a measure of the error in prediction, is the difference between the actual $y$-value ($y$) and the predicted $y$-value ($\hat{y}$). Residuals are represented on the graph by the vertical distance between a data point and

<table>
<thead>
<tr>
<th>Year (0 = 1990)</th>
<th>Tuition Rate</th>
<th>Predicted Rate</th>
<th>Residuals</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6546</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>6996</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>6996</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>7350</td>
<td></td>
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</tr>
<tr>
<td>4</td>
<td>7500</td>
<td></td>
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<tr>
<td>5</td>
<td>7978</td>
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<td>6</td>
<td>8377</td>
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<td>7</td>
<td>8710</td>
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<td>8</td>
<td>9110</td>
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<tr>
<td>9</td>
<td>9411</td>
<td></td>
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</tr>
<tr>
<td>10</td>
<td>9800</td>
<td></td>
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</tbody>
</table>
A residual plot is a graph that shows the residuals on the vertical axis and the independent variable on the horizontal axis. If the points in a residual plot are randomly dispersed around the horizontal axis, a linear regression model is appropriate for the data; otherwise, a non-linear model is more appropriate.

Students can determine the residual for any value in a data set.

Example: The table to the left displays the annual tuition rates of a state college in the U.S. between 1990 and 2000, inclusively. The linear function $R(t) = 326x + 6440$ has been suggested as a good fit for the data.

a) Extend the table to find the predicted rates based on the model and the residual values for each year.
b) Create the residual plot for the tuition rates.
c) Use the residual plot to determine the goodness of fit of the function for the data provided in the table.

Students can use a residual plot to determine the appropriateness of a linear model for a set of data.

Example: What do the following residual plots tell you about the appropriateness of a linear model for the functions they represent? Explain your responses.
NC Math 1
Unit 3: Introduction to Exponential Functions

RESEARCH BRIEF: **Unit 3: Exponential Functions**

**Essential Questions:**
- What are the basic patterns of exponential growth?
- How can the patterns of exponential growth be expressed with symbolic rules?
- What are the recursive and explicit forms for basic exponential functions?
- How are geometric sequences related to exponential functions?
- How can you represent and reason about functions involved in investments paying compound interest?
- What are some useful strategies for finding functions modeling patterns of change that are only approximately exponential?
- What mathematical patterns in tables, graphs, and symbolic rules are typical of exponential decay relations?
- How can you interpret parts of exponential growth and decay expressions in context of the scenario?
- How can logical analysis of an experiment be used as a check of a function model produced by your calculator or computer curve-fitting software?

**Learning Outcomes**
- Students will recognize patterns of exponential growth and decay from multiple representations.

**Student Objectives**
- I will **explain** an exponential function using multiple representations as a model of growth or decay.
● Students will use the standard rules of exponents to write exponential expressions in equivalent forms.
● Students will model exponential functions using rules in the form \( y = a(b^x) \) where \( a > 0 \) and \( b > 1 \) or \( 0 < b < 1 \).
● Students should understand the key features of an exponential function’s table, graph, and verbal representation.
● Students should recognize the difference between factor and rate.
● Students can use the increasing rate of change to examine the difference between the end behavior of linear and exponential functions.
● Students will calculate the average rate of change over a specified interval and differentiate between linear and exponential.
● Students translate between explicit and recursive forms of geometric sequences and use both to model situations.
● Students can represent and reason about functions involved in exponential growth and decay situations.
● Students can use technology to solve and understand solutions of exponential equations and inequalities.
● Students can use appropriate technology to calculate exponential regression for a scatterplot or a given set of data.
● Students will use exponential regression to predict outcomes from a given value.

● I will apply the properties of exponents to write expressions in equivalent forms. \( \text{NC.M1.N-RN.2} \)
● I will use the standard form of an exponential function to graphically represent a translation between pairs of functions. \( \text{NC.M1.F-LE.5} \)
● I will calculate the average rate of change over a specified interval of a function. \( \text{NC.M1. F-IF.6} \)
● I will calculate the rate of change to analyze the end behavior of an exponential function. \( \text{NC.M1.F-IF.6, NC.M1.F-LE.3} \)
● I will identify situations as linear or exponential based on average rate of change. \( \text{NC.M1.F-LE.3} \)
● I will recognize the difference between factor and rate in an exponential functions. \( \text{NC.M1.F-LE.5} \)
● I will develop explicit and recursive forms of geometric sequences to model a scenario. \( \text{NC.M1.F-IF.3, NC.M1.F-BF.2} \)
● Given multiple representations, I will create a function for exponential growth and decay situations. \( \text{NC.M1.A-CED.1,NC.M1.A-CED.2, NC.M1.F-BF.1a, NC.M1.F-BF.1b} \)
● I will explain the solutions from exponential equations and inequalities in the context of a problem. \( \text{NC.M1.A.REI.10} \)
● I will use technology to calculate the exponential regression from a set of data and predict other values. \( \text{NC.M1.S-ID.6c} \)
# Standards Addressed in this Unit

**Understand and apply the properties of exponents.**

<table>
<thead>
<tr>
<th>OCS Priority Standard(s)</th>
<th>Supporting Standard(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>NC.M1.N-RN.2</strong>: Extend the properties of exponents. Rewrite algebraic expressions with integer exponents using the properties of exponents.</td>
</tr>
</tbody>
</table>

**Determine the explicit and recursive formula for given geometric sequences.**

<table>
<thead>
<tr>
<th>OCS Priority Standard(s)</th>
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</thead>
<tbody>
<tr>
<td></td>
<td><strong>NC.M1.F-IF.3</strong>: Recognize that recursively and explicitly defined sequences are functions whose domain is a subset of the integers, the terms of an arithmetic sequence are a subset of the range of a linear function, and the terms of a geometric sequence are a subset of the range of an exponential function. <strong>NC.M1.F-BF.2</strong>: Translate between explicit and recursive forms of arithmetic and geometric sequences and use both to model situations.</td>
</tr>
</tbody>
</table>

**Evaluate, create, and interpret exponential functions in context.**

<table>
<thead>
<tr>
<th>OCS Priority Standard(s)</th>
<th>Supporting Standard(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>NC.M1.F-IF.4</strong>: Interpret functions that arise in applications in terms of the context. Interpret key features of graphs, tables, and verbal descriptions in context to describe functions that arise in applications relating two quantities, including:</td>
<td><strong>NC.M1.F-IF.2</strong>: Understand the concepts of a functions and use function notation. Use function notation to evaluate linear, quadratic, and exponential functions for inputs in their domains, and interpret statements that use function notation in terms of a context.</td>
</tr>
</tbody>
</table>
intercepts; intervals where the function is increasing, decreasing, positive, or negative; and maximums and minimums.

**NC.M1.A-CED.1:** Create equations that describe numbers or relationships. Create equations and inequalities in one variable that represent linear, exponential, and quadratic relationships and use them to solve problems.

**NC.M1.A-CED.2:** Create equations that describe numbers or relationships. Create and graph equations in two variables to represent linear, exponential, and quadratic relationships between quantities.

**NC.M1.A-REI.10:** Represent and solve equations and inequalities graphically. Understand that the graph of a two variable equation represents the set of all solutions to the equation.

**NC.M1.F-LE.5:** Interpret expressions for functions in terms of the situation they model. Interpret the parameters a and b in a linear function \(f(x)=ax+b\) or an exponential function \(g(x)=ab^x\) in terms of a context.

**NC.M1.F-IF.6:** Interpret functions that arise in applications in terms of the context. Calculate and interpret the average rate of change over a specified interval for a function presented numerically, graphically, and/or symbolically.

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**Identify situations and practical domains for exponential functions.**

<table>
<thead>
<tr>
<th>OCS Priority Standard(s)</th>
<th>Supporting Standard(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>NC.M1.F-IF.7:</strong> Analyze linear, exponential, and quadratic functions by generating different representations, by hand in simple cases and using technology for more complicated cases, to show key features, including: domain and range; rate of change; intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximums and minimums; and end behavior.</td>
<td><strong>NC.M1.F-LE.1:</strong> Identify situations that can be modeled with linear and exponential functions, and justify the most appropriate model for a situation based on the rate of change over equal intervals. <strong>NC.M1.F-IF.5:</strong> Interpret a function in terms of context by relating its domain and range to its graph and, where applicable, to the quantitative relationship it describes.</td>
</tr>
</tbody>
</table>
Compare, interpret, and explain key features of exponential functions.

<table>
<thead>
<tr>
<th>OCS Priority Standard(s)</th>
<th>Supporting Standard(s)</th>
</tr>
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<tbody>
<tr>
<td><strong>NC.M1.A-SSE.1a</strong>: Interpret the structure of expressions. Identify and interpret parts of a linear, exponential, or quadratic expression, including terms, factors, coefficients, and exponents. <strong>NC.M1.F-IF.9</strong>: Compare key features of two functions (linear, quadratic, or exponential) each with a different representation (symbolically, graphically, numerically in tables, or by verbal descriptions).</td>
<td><strong>NC.M1.F-IF.8b</strong>: Analyze functions using different representations. Interpret and explain growth and decay rates for an exponential function. <strong>NC.M1.F-LE.3</strong>: Construct and compare linear and exponential models and solve problems. Compare the end behavior of linear, exponential, and quadratic functions using graphs and tables to show that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically. <strong>NC.M1.A-REL.11</strong>: Build an understanding of why the x-coordinates of the points where the graphs of two linear, exponential, or quadratic equations and intersect are the solutions of the equation and approximate solutions using a graphing technology or successive approximations with a table of values.</td>
</tr>
</tbody>
</table>

Write and apply exponential functions given multiple representations.

<table>
<thead>
<tr>
<th>OCS Priority Standard(s)</th>
<th>Supporting Standard(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>NC.M1.S.ID.6c</strong>: Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Fit a function to exponential data using technology. Use the fitted function to solve problems.</td>
<td><strong>NC.M1.F-BF.1a</strong>: Build linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two ordered pairs (include reading these from a table). <strong>NC.M1.F-BF.1b</strong>: Build a function that models a relationship between two quantities by combining linear, exponential, or quadratic functions with addition and subtraction or two linear functions with multiplication.</td>
</tr>
</tbody>
</table>
NC.M1.N-RN.2

*Extend the properties of exponents.*
Rewrite algebraic expressions with integer exponents using the properties of exponents.

### Concepts and Skills

**Pre-requisite**
- Using the properties of exponents to create equivalent numerical expressions (8.EE.1)

**Connections**
- Use operations to rewrite polynomial expressions (NC.M1.A-APR.1)

### The Standards for Mathematical Practices

**Implementing the Standards for Mathematical Practice**

**Connections**

*Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.*

7 – Look for and make use of structure
8 – Look for and express regularity in repeated reasoning

**Disciplinary Literacy**

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

- Vocabulary – base, exponent, index
- Students should be able to justify their steps in rewriting algebraic expressions.

### Mastering the Standard

<table>
<thead>
<tr>
<th>Comprehending the Standard</th>
<th>Assessing for Understanding</th>
</tr>
</thead>
</table>
| Students extend the properties of integer exponents learned in middle school with numerical expressions to algebraic expressions. The process of “simplifying square roots” is not an expectation for Math 1 students. In Math 2, students will extend the properties of exponents to rational exponents and rewrite, “simplify” all square roots. | Students should be able to use the properties of exponents to write expression into equivalent forms. **Example:** Rewrite the following with positive exponents:

  a) \((8x^{-4}y^3)(-2x^5y^{-6})^2\)

  b) \(\frac{(3m^2p^{-2}q)^3}{9m^{-3}q^2}\)

Students should be able to use the new skills of applying the properties of exponents with skills learned in previous courses. **Example:** Simplify: \(\sqrt{25m^{14}p^{2}q^{4}}\)

*In 8th grade, students learned to evaluate the square roots of perfect squares and the cube root of perfect cubes. In Math 1, students can combine this previous skill with algebraic expressions. When addressing a problem like this in Math 1, students should be taught to rewrite the expression using the properties of exponents and then using inverse operations to rewrite.*

**For example,** \(\sqrt{m^{14}} = \sqrt{(m^7)^2} = m^7.\)
NC.M1.F-IF.3

**Understand the concept of a function and use function notation.**
Recognize that recursively and explicitly defined sequences are functions whose domain is a subset of the integers, the terms of an arithmetic sequence are a subset of the range of a linear function, and the terms of a geometric sequence are a subset of the range of an exponential function.

<table>
<thead>
<tr>
<th>Concepts and Skills</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pre-requisite</strong></td>
</tr>
<tr>
<td>● Interpret the equation ( y = mx + b ) as being from a linear function and compare to nonlinear functions (8.F.3)</td>
</tr>
<tr>
<td>● Define a function and use functions notation (NC.M1.F-IF.1)</td>
</tr>
<tr>
<td>● Evaluating functions (NC.M1.F-IF.2)</td>
</tr>
<tr>
<td><strong>Connections</strong></td>
</tr>
<tr>
<td>● Relating the domain and range to a context (NC.M1.F-IF.5)</td>
</tr>
<tr>
<td>● Analyzing linear and exponential functions (NC.M1.F-IF.7)</td>
</tr>
<tr>
<td>● Build linear and exponential functions (NC.M1.F-BF.1)</td>
</tr>
<tr>
<td>● Translate between explicit and recursive forms (NC.M1.F-BF.2)</td>
</tr>
<tr>
<td>● Identify situations that can be modeled with linear and exponential functions (NC.M1.F-LE.1)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>The Standards for Mathematical Practices</th>
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</thead>
<tbody>
<tr>
<td><strong>Implementing the Standards for Mathematical Practice</strong></td>
</tr>
<tr>
<td><strong>Connections</strong></td>
</tr>
<tr>
<td><em>Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.</em></td>
</tr>
<tr>
<td>8 – Look for and express regularity in repeated reasoning</td>
</tr>
<tr>
<td><strong>Disciplinary Literacy</strong></td>
</tr>
<tr>
<td><em>As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.</em></td>
</tr>
<tr>
<td>Students should be able to explain a function written in recursive form using subset notation.</td>
</tr>
<tr>
<td>New Vocabulary: arithmetic sequence, geometric sequence, explicit form, recursive form, exponential function</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mastering the Standard</th>
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</thead>
<tbody>
<tr>
<td><strong>Comprehending the Standard</strong></td>
</tr>
<tr>
<td>Students should recognize that sequences are functions. A sequence can be described as a function, with the domain consisting of a subset of the integers, and the range being the terms of the sequence. This standard connects to arithmetic and geometric sequences and should be taught with NC.M1.F-BF.2. Emphasize that arithmetic and geometric sequences are examples of linear and exponential functions, respectively. It is important to note that sequences are not limited to</td>
</tr>
<tr>
<td><strong>Assessing for Understanding</strong></td>
</tr>
<tr>
<td>Example: A theater has 60 seats in the first row, 68 seats in the second row, 76 seats in the third row, and so on in the same increasing pattern.</td>
</tr>
<tr>
<td>a) If the theater has 20 rows of seats, how many seats are in the twentieth row?</td>
</tr>
<tr>
<td>b) Explain why the sequence is considered a function.</td>
</tr>
<tr>
<td>c) What is the domain of the sequence? Explain what the domain represents in context.</td>
</tr>
<tr>
<td>d) What is the range of the sequence? Explain what the range represents in context.</td>
</tr>
<tr>
<td>Example: A geometric sequence can be represented by the exponential function ( f(x) = 400 \left( \frac{1}{2} \right)^x ). In terms of the geometric sequence, explain what ( f(3) = 50 ) represents.</td>
</tr>
</tbody>
</table>
arithmetic and geometric. It is expected that recursive form should be written in subset notation. Students should be familiar with writing and interpreting subset notation.
Now-Next can be used as a tool to introduce the concepts of recursive form, but the expectation is that students will move to the more formal representations of recursive form.

**Example:** Represent the following sequence in explicit form: 1, 4, 9, 16, 25

**Example:** The Fibonacci numbers are a sequence that are often found in nature. This sequence is defined by $a_n = a_{n-1} + a_{n-2}$ where $a_0 = 0$ and $a_1 = 1$. What are the first 10 terms of the Fibonacci sequence? Could you easily represent this pattern in explicit form?

**NC.M1.F-BF.2**

*Build a function that models a relationship between two quantities.*
Translate between explicit and recursive forms of arithmetic and geometric sequences and use both to model situations.

## Concepts and Skills

<table>
<thead>
<tr>
<th>Pre-requisite</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Construct a function to model a linear relationship (8.F.4)</td>
</tr>
<tr>
<td>• Formally define a function (NC.M1.F-IF.1)</td>
</tr>
<tr>
<td>• Recognize sequences as function and link arithmetic sequences to linear functions and geometric sequences to exponential functions (NC.M1.F-IF.3)</td>
</tr>
<tr>
<td>• Build functions from arithmetic and geometric sequences (NC.M1.F-BF.1a)</td>
</tr>
</tbody>
</table>

## The Standards for Mathematical Practices

### Implementing the Standards for Mathematical Practice

**Connections**

*Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.*

*4 – Model with mathematics*

**Disciplinary Literacy**

*As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.*

Students should be able to explain their model in context.
New Vocabulary: arithmetic sequence, geometric sequence, explicit form, recursive form

### Mastering the Standard

<table>
<thead>
<tr>
<th>Comprehending the Standard</th>
<th>Assessing for Understanding</th>
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</thead>
</table>
| Students should be able to use both the explicit and recursive forms of arithmetic and geometric sequences where the explicit form is a linear or exponential function, respectively. | Students should be able to build explicit and recursive forms of arithmetic and geometric sequences. **Example:** The sequence below shows the number of trees that a nursery plants each year. $2, 8, 32, 128 ...$
Let $a_n$ represent the current term in the sequence and $a_{n-1}$ represent the previous term in the sequence. Which formula could be used to determine the number of trees the nursery will plant in year $n$? |
Students are expected to use formal notation:
- $a_n$ (NOW)
- $a_{n-1}$ (PREVIOUS)
- $a_{n+1}$ (NEXT)
(Students can use NEXT-NOW notation as they learn to create recursive functions but will need to move to formal notation.)

This standard should be tied to NC.M1.F-IF.3, recognizing patterns and linking to function types.

Students should recognize explicit form of an arithmetic sequence as an equivalent structure to slope-intercept form of a linear function and explicit form of a geometric sequence as an equivalent structure to standard form of an exponential function. Using the concepts of rate of change, students should recognize that the forms of these sequences are one iteration forward from the $y$-intercept, which gives meaning to the $n - 1$ notation.

A) $a_n = 4a_{n-1}$
B) $a_n = \frac{1}{4}a_{n-1}$
C) $a_n = 2a_{n-1} + 4$
D) $a_n = a_{n-1} + 6$

Example: A single bacterium is placed in a test tube and splits in two after one minute. After two minutes, the resulting two bacteria split in two, creating four bacteria. This process continues.

a) How many bacteria are in the test tube after 5 minutes? 15 minutes?
b) Write a recursive rule to find the number of bacteria in the test tube from one minute to the next.
c) Convert this rule into explicit form. How many bacteria are in the test tube after one hour?

Example: A concert hall has 58 seats in Row 1, 62 seats in Row 2, 66 seats in Row 3, and so on. The concert hall has 34 rows of seats.

a) Write a recursive formula to find the number of seats in each row. How many seats are in row 5?
b) Write the explicit formula to determine which row has 94 seats?

Example: Given the sequence defined by the function $a_{n+1} = a_n + 12$ with $a_1 = 4$. Write an explicit function rule.

Note: Student may interpret 4 as the $y$-intercept since it is the first value; however, attending to the notation when $x = 1, y = 4$. Thus, the $y$-intercept for the explicit form is -8.

Example: Given the sequence defined by the function $a_{n+1} = \frac{3}{4}a_n$ with $a_1 = 424$. Write an explicit function rule.
NC.M1.F-IF.2

*Understand the concept of a function and use function notation.*

Use function notation to evaluate linear, quadratic, and exponential functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

<table>
<thead>
<tr>
<th>Pre-requisite</th>
</tr>
</thead>
<tbody>
<tr>
<td>● Use substitution to determine if a number if a solution (6.EE.5)</td>
</tr>
<tr>
<td>● Interpret parts of expressions in context (NC.M1.A-SSE.1a, NC.M1.A-SSE.1b)</td>
</tr>
<tr>
<td>● Every point on the graph of an equation is a solution to the equation (NC.M1.A-REI.10)</td>
</tr>
<tr>
<td>● Define a function and use functions notation (NC.M1.F-IF.1)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Connections</th>
</tr>
</thead>
<tbody>
<tr>
<td>● Creating and solving one variable equations (NC.M1.A-CED.1)</td>
</tr>
<tr>
<td>● Creating and graphing two variable equations (NC.M1.A-CED.2)</td>
</tr>
<tr>
<td>● Every point on the graph of an equation is a solution to the equation (NC.M1.A-REI.10)</td>
</tr>
<tr>
<td>● Comparing the end behavior of functions (NC.M1.F-LE.3)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>The Standards for Mathematical Practices</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Implementing the Standards for Mathematical Practice</strong></td>
</tr>
</tbody>
</table>

**Connections**

*Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.*

**Disciplinary Literacy**

*As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.*

Students should be able to discuss the domain, range, input, output and the relationship between the variables of a function in context.

**New Vocabulary:** exponential function, quadratic function

<table>
<thead>
<tr>
<th>Comprehending the Standard</th>
<th>Assessing for Understanding</th>
</tr>
</thead>
</table>
| Students should be fluent in using function notation to evaluate a linear, quadratic, and exponential function. Students should be able to interpret statements in function notation in contextual situations. | Students should be able to use evaluate functions written in function notation. **Example:** Evaluate $f(2)$ for the function $f(x) = 5(x - 3) + 17$. Evaluate $f(2)$ for the function $f(x) = 1200(1 + .04)^x$. Evaluate $f(2)$ for the function $f(x) = 3x^2 + 2x - 5$. Students should be able to evaluate functions and interpret the result in a context. **Example:** You placed a yam in the oven and, after 45 minutes, you take it out. Let $f$ be the function that assigns to each minute after you placed the yam in the oven, its temperature in degrees Fahrenheit. Write a sentence for each of the
following to explain what it means in everyday language.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>( f(0) = 65 )</td>
</tr>
<tr>
<td>b)</td>
<td>( f(5) &lt; f(10) )</td>
</tr>
<tr>
<td>c)</td>
<td>( f(40) = f(45) )</td>
</tr>
<tr>
<td>d)</td>
<td>( f(45) &gt; f(60) )</td>
</tr>
</tbody>
</table>

(https://www.illustrativemathematics.org/content-standards/HSF/IF/A/2/tasks/625)

Example: The rule \( f(x) = 50(0.85)^x \) represents the amount of a drug in milligrams, \( f(x) \), which remains in the bloodstream after \( x \) hours. Evaluate and interpret each of the following:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>( f(0) )</td>
</tr>
<tr>
<td>b)</td>
<td>( f(2) = k \cdot f(1) ). What is the value of ( k )?</td>
</tr>
<tr>
<td>c)</td>
<td>( f(x) &lt; 6 )</td>
</tr>
</tbody>
</table>

Example: Suppose that the function \( f(x) = 2x + 12 \) represents the cost to rent \( x \) movies a month from an internet movie club. Makayla now has $10. How many more dollars does Makayla need to rent 7 movies next month?

(NCDPI Math 1 released EOC #12)

Example: Let \( f(t) \) be the number of people, in millions, who own cell phones \( t \) years after 1990. Explain the meaning of the following statements.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>( f(10) = 100.3 )</td>
</tr>
<tr>
<td>b)</td>
<td>( f(a) = 20 )</td>
</tr>
<tr>
<td>c)</td>
<td>( f(20) = b )</td>
</tr>
<tr>
<td>d)</td>
<td>( n = f(t) )</td>
</tr>
</tbody>
</table>

(https://www.illustrativemathematics.org/content-standards/HSF/IF/A/2/tasks/634)
NC.M1.F-IF.4  
*Interpret functions that arise in applications in terms of the context.*
Interpret key features of graphs, tables, and verbal descriptions in context to describe functions that arise in applications relating two quantities, including: intercepts; intervals where the function is increasing, decreasing, positive, or negative; and maximums and minimums.

### Concepts and Skills

#### Pre-requisite
- Describe quantitatively the functional relationship between two quantities by analyzing a graph (8.F.5)
- Define a function and use functions notation (NC.M1.F-IF.1)
- Evaluating functions (NC.M1.F-IF.2)

#### Connections
- Interpret parts of expressions in context (NC.M1.A-SSE.1a, NC.M1.A-SSE.1b)
- Relate domain and range of a function to its graph (NC.M1.F-IF.5)
- Calculate the average rate of change (NC.M1.F-IF.6)
- Use equivalent forms of quadratic and exponential function to reveal key features (NC.M1.F-IF.8a, NC.M1.F-IF.8b)
- Compare key features of two functions in different representations (NC.M1.F-IF.9)
- Identify situations that can be modeled with linear and exponential functions (NC.M1.F-LE.1)

### The Standards for Mathematical Practices

#### Implementing the Standards for Mathematical Practice

#### Connections
*Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.*

4 – Model with mathematics

#### Disciplinary Literacy
*As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.*

Students should be able to justify their identification of key features and interpret those key features in context.

**New Vocabulary:** maximum, minimum

### Mastering the Standard

#### Comprehending the Standard
Students should understand the key features of any contextual situation. For example, plots over time represent functions as do some scatterplots. These are often functions that "tell a story" hence the portion of the standard that has students sketching graphs given a verbal description. Students should have experience

#### Assessing for Understanding
Students should be able to identify and interpret key features of functions.

**Example:** An epidemic of influenza spreads through a city. The figure below is the graph of \( I = f(w) \), where \( I \) is the number of individuals (in thousands) infected \( w \) weeks after the epidemic begins.
with a wide variety of these types of functions and be flexible in thinking about functions and key features using tables, graphs, and verbal descriptions. Students should understand the concept behind the key features (intercepts, increasing/decreasing, positive/negative, and maximum/minimum) for any given graph, not just “function families”. This means that students should be asked to work with graphical and tabular representations of functions that the student could not solve or manipulation algebraically.

By contrast, NC.M1.F-IF.7, has students work with specific functions in which students have the ability to use algebraic manipulation to identify additional key features.

a) Estimate \( f(2) \) and explain its meaning in terms of the epidemic.
b) Approximately how many people were infected at the height of the epidemic? When did that occur? Write your answer in the form \( f(a) = b \).
c) For approximately which \( w \) is \( f(w) = 4.5 \); explain what the estimates mean in terms of the epidemic.
d) An equation for the function used to plot the image above is \( f(w) = 6w(1.3)^{-w} \). Use the graph to estimate the solution of the inequality \( 6w(1.3)^{-w} \geq 6 \). Explain what the solution means in terms of the epidemic. (This would make a great Honors level extension to this standard)

Example: The figure shows the graph of \( T \), the temperature (in degrees Fahrenheit) over one particular 20-hour period in Santa Elena as a function of time \( t \).

<table>
<thead>
<tr>
<th>Time (minutes)</th>
<th>Distance (feet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>30</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
</tr>
<tr>
<td>4</td>
<td>25</td>
</tr>
<tr>
<td>5</td>
<td>50</td>
</tr>
</tbody>
</table>

Example: Eliana observed her dog, Lola, running around the yard and recorded the time and distance that Lola was away from her dog house in the table below.
a) Sketch a graph of Lola’s play time away from her dog house.
b) Describe what is happening between minutes 2 & 3.
NC.M1.A-CED.1

*Create equations that describe numbers or relationships.*

Create equations and inequalities in one variable that represent linear, exponential, and quadratic relationships and use them to solve problems.

<table>
<thead>
<tr>
<th>Concepts and Skills</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pre-requisite</strong></td>
</tr>
<tr>
<td>● Create two-step linear equations and inequalities from a context (7.EE.4)</td>
</tr>
<tr>
<td><strong>Connections</strong></td>
</tr>
<tr>
<td>● Interpret parts of an expression in context (NC.M1.A-SSE.1a,b)</td>
</tr>
<tr>
<td>● Justify a chosen solution method and each step of a that process (NC.M1.A-REI.1)</td>
</tr>
<tr>
<td>● Solve linear, exponential and quadratic equations using tables and graphs (NC.M1.A-REI.11)</td>
</tr>
<tr>
<td>● Represent the solutions of linear inequalities on a graph (NC.M1.A-REI.12)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>The Standards for Mathematical Practices</th>
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</thead>
<tbody>
<tr>
<td><strong>Implementing the Standards for Mathematical Practice</strong></td>
</tr>
<tr>
<td><strong>Connections</strong></td>
</tr>
<tr>
<td>Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.</td>
</tr>
<tr>
<td>4 – Model with mathematics</td>
</tr>
<tr>
<td><strong>Disciplinary Literacy</strong></td>
</tr>
<tr>
<td>As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.</td>
</tr>
<tr>
<td>Students should be able to describe the origins of created equations and inequalities and demonstrate its relation to the context.</td>
</tr>
<tr>
<td>New Vocabulary: exponential function, quadratic function</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mastering the Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Comprehending the Standard</strong></td>
</tr>
<tr>
<td>Students create equations and inequalities in one-variable and use them to solve the problems.</td>
</tr>
<tr>
<td>In Math I, focus on linear, quadratic, and exponential contextual situations that students can use to create equations and inequalities in one variable and use them to solve problems. It is also important to note that equations can also be created from an associated function. After the students have created an equation, they can use other representations to solve</td>
</tr>
<tr>
<td><strong>Assessing for Understanding</strong></td>
</tr>
<tr>
<td>Students should be able to create an equation from a function and use the equation to solve problems.</td>
</tr>
<tr>
<td>Students should be able to create equations from various representations, such as verbal descriptions, and use them to solve problems.</td>
</tr>
<tr>
<td><strong>Example:</strong> Phil purchases a used truck for $11,500. The value of the truck is expected to decrease by 20% each year. When will the truck first be worth less than $1,000?</td>
</tr>
<tr>
<td><strong>Example:</strong> Suppose a friend tells you she paid a total of $16,368 for a car, and you'd like to know the car's list price (the price before taxes) so that you can compare prices at various dealers. Find the list price of the car if your friend bought the car in:</td>
</tr>
<tr>
<td>a) Arizona, where the sales tax is 6.6%.</td>
</tr>
</tbody>
</table>
problems, such as graphs and tables.

For quadratic and exponential inequalities, the focus of this standard is to create the inequality and use that inequality to solve a problem. Solving these inequalities algebraically is NOT part of the standard. Once a student has the inequality, the student can use a table or graph to find a solution to the problem.

Students in Math I are not responsible for interval notation as a solution. They are to write answers to these inequalities using inequality notation.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>b)</td>
<td>New York, where the sales tax is 8.25%.</td>
</tr>
<tr>
<td>c)</td>
<td>A state where the sales tax is ( r ).</td>
</tr>
</tbody>
</table>

(https://www.illustrativemathematics.org/content-standards/HSA/CED/A/1/tasks/582)

Students should be able to create inequalities and use those inequalities to solve problems. (Students are not expected to solve quadratic and exponential inequalities algebraically. Students should use technology, tables and graphs to solve problems.)

**Example:** Susanna heard some exciting news about a well-known celebrity. Within a day she told 4 friends who hadn’t heard the news yet. By the next day each of those friends told 4 other people who also hadn’t yet heard the news. By the next day each of those people told four more, and so on.

a) Assume the rumor continues to spread in this manner. Let \( N \) be the function that assigns to \( d \) the number of people who hear the rumor on the \( d \)th day. Write an expression for \( N(d) \).

b) On which day will at least 100,000 people hear the rumor for the first time?

c) How many people will hear the rumor for the first time on the 20th day?

d) Is the answer to (c) realistic? Explain your reasoning.

e) Create an inequality that could be used to determine when there will be greater than 200,000 people that have heard the rumor.

https://www.illustrativemathematics.org/content-standards/HSF/LE/A/2/tasks/74
NC.M1.A-CED.2

Create equations that describe numbers or relationships.
Create and graph equations in two variables to represent linear, exponential, and quadratic relationships between quantities.

Pre-requisite

- Construct a linear function that models the relationship between two quantities (8.F.4)
- Graph linear equations (8.EE.6)
- The graph of a function is the set of ordered pairs consisting of input and a corresponding output (8.F.1)
- Understand that the graph of a two variable equation represents the set of all solutions to the equation (NC.M1.A-REI.10)

Connections

- Interpret parts of an expression in context (NC.M1.A-SSE.1a,b)
- Creating linear equations for a system (NC.M1.A-CED.3)
- Solving for a variable of interest in a formula (NC.M1.A-CED.4)
- The graph a function \( f \) is the graph of the equation \( y = f(x) \) (NC.M1.F-IF.1)
- Interpret a function’s domain and range in context (NC.M1.F-IF.5)
- Identify key features of linear, exponential and quadratic functions (NC.M1.F-IF.7)
- Building a function through patterns or by combining other functions (NC.M1.F-BF.1a, NC.M1.F-BF.1b)

The Standards for Mathematical Practices

Implementing the Standards for Mathematical Practice

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.
4 – Model with mathematics
6 – Attend to precision

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to describe the origins of created equations and demonstrate its relation to the context.

New Vocabulary: exponential function, quadratic function

Mastering the Standard

<table>
<thead>
<tr>
<th>Comprehending the Standard</th>
<th>Assessing for Understanding</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students create equations in two variables. Graph equations on coordinate axes with labels and scales clearly labeling the axes defining what the values on the axes represent and the unit of measure. Students also select intervals for the scale that are appropriate for</td>
<td>Students should be able to create two variable equations from various representations, such as verbal descriptions, and use them to solve problems. Example: The larger leg of a right triangle is 3 cm longer than its smaller leg. The hypotenuse is 6 cm longer than the smaller leg. How many centimeters long is the smaller leg? (NCDPI Math 1 released EOC #13) Example: The floor of a rectangular cage has a length 4 feet greater than its width, w. James will increase both</td>
</tr>
</tbody>
</table>

September 2017 Update - Feedback NC Math 1 Unit 3 Introduction to Exponential Functions Return to: Standards Addressed in this Unit
the context and display adequate information about the relationship.
Students interpret the context and choose appropriate minimum and maximum values for a graph.
In Math I, focus on linear, exponential and quadratic **contextual** situations for students to create equations in two variables.
While students will one be asked to rewrite expressions with integers exponents, in exponential functions, the domain is not restricted and students should use technology to see the continuity of exponential functions.

Dimensions of the floor by 2 feet. Which equation represents the new area, $N$, of the floor of the cage?

A) $N = w^2 + 4w$
B) $N = w^2 + 6w$
C) $N = w^2 + 6w + 8$
D) $N = w^2 + 8w + 12$

(НСDPI Math I released EOC #5)

Students should be able to create a two variable equations, graph the relationship, and use graph to recognize key feature of the graph.

**Example:** The FFA had a fundraiser by selling hot dogs for $1.50 and drinks for $2.00. Their total sales were $400.

a) Write an equation to calculate the total of $400 based on the hot dog and drink sales.

b) Graph the relationship between hot dog sales and drink sales.

*Note: This make a good connection to NC.M1.F-IF.5*

**Example:** In a women's professional tennis tournament, the money a player wins depends on her finishing place in the standings. The first-place finisher wins half of $1,500,000 in total prize money. The second-place finisher wins half of what is left; then the third-place finisher wins half of that, and so on.

a) Write a rule to calculate the actual prize money in dollars won by the player finishing in nth place, for any positive integer n.

b) Graph the relationship between the first 10 finishers and the prize money in dollars.

What pattern is indicated in the graph? What type of relationship exists between the two variables?
NC.M1.A-REI.10

**Represent and solve equations and inequalities graphically**
Understand that the graph of a two variable equation represents the set of all solutions to the equation.

<table>
<thead>
<tr>
<th>Pre-requisite</th>
</tr>
</thead>
<tbody>
<tr>
<td>● Use substitution to determine if a number is a solution (6.EE.5)</td>
</tr>
<tr>
<td>● Graphing lines (8.EE.5, 8.EE.6, 8.F.3)</td>
</tr>
<tr>
<td>● Analyze and solve pairs of simultaneous linear equations by graphing and substitution (8.EE.8)</td>
</tr>
<tr>
<td>● Understanding functions as a rule that assigns each input with exactly one output (8.F.1)</td>
</tr>
</tbody>
</table>

**Connections**

- Creating and graphing two-variable equations (NC.M1.A-CED.2)
- Understanding that the relationship between the solution of system of equations and the associated equation (NC.M1.A-REI.11)
- Representing the solutions to linear inequalities (NC.M1.A-REI.12)
- Relating a function to its graph, domain and range of a function (NC.M1.F-IF.1, NC.M1.F-IF.2, NC.M1.F-IF.5)

**The Standards for Mathematical Practices**

**Implementing the Standards for Mathematical Practice**

**Connections**

*Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.*

**Disciplinary Literacy**

*As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.*

Students should be able to discuss the solutions to a two variable equation and the link to a function.

**Mastering the Standard**

<table>
<thead>
<tr>
<th>Comprehending the Standard</th>
<th>Assessing for Understanding</th>
</tr>
</thead>
</table>
| Students understand that the graph of an equation is the set of all ordered pairs that make that equation a true statement. This standard contains no limitation and so applies to all function types, including those functions that a student cannot yet algebraically manipulate. | Students should be able to assess if a point is a solution to an equation. **Example:** Consider three points in the plane, \( P = (-4,0) \), \( Q = (-1,12) \) and \( R = (4,32) \).  
   a) Find the equation of the line through \( P \) and \( Q \).  
   b) Use your equation in (a) to show that \( R \) is on the same line as \( P \) and \( Q \).  
   ([https://www.illustrativemathematics.org/content-standards/HSA/REI/D/10/tasks/1066](https://www.illustrativemathematics.org/content-standards/HSA/REI/D/10/tasks/1066))  

**Example:** Which of the following points are on the graph of the equation \(-5x + 2y = 20\)? Which of the following points are of the graph of the equation? How do you know?  
   a) \((4,0)\)
<table>
<thead>
<tr>
<th>Students can explain and verify that every point ((x, y)) on the graph of an equation represents all values for (x) and (y) that make the equation true.</th>
</tr>
</thead>
</table>
| b) \((0, 10)\)  
| c) \((-1, 7.5)\)  
| d) \((2.3, 5)\)  |

**Example:** Verify that \((-1, 60)\) is a solution to the equation \(y = 15\left(\frac{1}{2}\right)^x\). Explain what this means for the graph of the function.

**Example:** Without graphing, determine if the ordered pair \((2, -15)\) is on the graph of \(y = 3x^2 + 2x - 1\). Explain.

**Example:** The graph to the right shows the height of a hot air balloon as a function of time.

Use the graph to answer the following:

a) What is the height of the hot air balloon 10 minutes after it has left the ground?  
b) Approximately, when will the hot air balloon reach a height of 600 feet?  
c) Explain what the point \((48, 800)\) on this graph represents.
NC.M1.F-LE.5
Interpret expressions for functions in terms of the situation they model.
Interpret the parameters $a$ and $b$ in a linear function $f(x) = ax + b$ or an exponential function $g(x) = ab^x$ in terms of a context.

<table>
<thead>
<tr>
<th>Concepts and Skills</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-requisite</td>
</tr>
<tr>
<td>- Construct a function to model a linear relationship and interpret rate of change and initial value (8.F.4)</td>
</tr>
<tr>
<td>- Compare the coefficients and constants of linear equations in similar form (8.EE.b)</td>
</tr>
<tr>
<td>- Identify and interpret parts of expression (NC.M1.A-SSE.1a, NC.M1.A-SSE.1b)</td>
</tr>
<tr>
<td>Connections</td>
</tr>
<tr>
<td>- Identify and interpret key features of functions from different representations (NC.M1.F-IF.7)</td>
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<td>Connections</td>
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<td>As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.</td>
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<tr>
<td>Comprehending the Standard</td>
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<tr>
<td>Students should know the meaning of the parameters in both linear and exponential functions in the context of the situation. Use real-world situations to help students understand how the parameters of linear and exponential functions depend on the context. In a linear function $y = ax + b$ the value of “$a$” represents the slope (constant rate of change) while “$b$” represents the $y$ intercept (initial value). In an exponential function $y = a(b)^x$ the value of “$a$” represents the $y$ intercept (initial value) and “$b$” represents the growth or decay factor.</td>
</tr>
<tr>
<td>Assessing for Understanding</td>
</tr>
<tr>
<td>Students should be able to describe the effects of changes to the parameters of a linear and exponential functions.</td>
</tr>
<tr>
<td><strong>Example:</strong> A plumber who charges $50 for a house call and $85 per hour can be expressed as the function $y = 85x + 50$. If the rate were raised to $90 per hour, how would the function change?</td>
</tr>
</tbody>
</table>
| **Example:** The equation $y = 8,000(1.04)^x$ models the rising population of a city with 8,000 residents when the annual growth rate is 4%.
  a) What would be the effect on the equation if the city’s population were 12,000 instead of 8,000? 
  b) What would happen to the population over 25 years if the growth rate were 6% instead of 4%? |
| Students should be able to interpret the parameters of a linear and exponential function. |
| **Example:** A function of the form $f(n)=P(1+r)^n$ is used to model the amount of money in a savings account that earns 8% interest, compounded annually, where $n$ is the number of years since the initial deposit.
  a) What is the value of $r$? Interpret what $r$ means in terms of the savings account? 
  b) What is the meaning of the constant $P$ in terms of the savings account? Explain your reasoning. |
When \( b > 1 \) the function models growth. When \( 0 < b < 1 \) the function models decay. Be cautious when interpreting the growth or decay rate. If the factor is 0.85 this means that it decreasing by 15%. If the factor is 1.05, this means that is increasing by 5

<table>
<thead>
<tr>
<th>c) Will ( n ) or ( f(n) ) ever take on the value 0? Why or why not?</th>
</tr>
</thead>
</table>

**Example:** Lauren keeps records of the distances and cost for her taxi rides.

a) If you graph the ordered pairs \((d, f)\) from the table, they lie on a line. How can this be determined without graphing them?

b) Show that the linear function in part a. has equation \( f = 2.25d + 1.5 \).

c) What do the 2.25 and the 1.5 in the equation represent in terms of taxi rides

---

### NC.M1.F-IF.6

*Interpret functions that arise in applications in terms of the context.*

Calculate and interpret the average rate of change over a specified interval for a function presented numerically, graphically, and/or symbolically.

---

### Concepts and Skills

**Pre-requisite**

- Determine and interpret the rate of change of a linear function (8.F.4)
- Describe qualitatively the functional relationship between two quantities and sketch a graph from a verbal description (8.F.5)

**Connections**

- Interpret key features of graphs and tables (NC.M1.F-IF.4)
- Analyze linear, quadratic and exponential functions by generating different representations (NC.M1.F-IF.7)

---

### Mastering the Standard

**Comprehending the Standard**

Students calculate the average rate of change of a function given a graph, table, and/or equation.

**The average rate of change of a function**

\[
\frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}
\]

**Assessing for Understanding**

**Example:** Find the average rate of change of each of the following functions over the interval \( 1 \leq x \leq 5 \).

a) \( f(x) = 3x - 7 \)

b) \( g(x) = x^2 + 2x - 5 \)

c) \( h(x) = 3(2)^x \)

**Example:** The table below shows the average weight of a type of plankton after several weeks.

<table>
<thead>
<tr>
<th>Distance ( d ) in miles</th>
<th>Fare ( f ) in dollars</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>8.25</td>
</tr>
<tr>
<td>5</td>
<td>12.75</td>
</tr>
<tr>
<td>11</td>
<td>26.25</td>
</tr>
</tbody>
</table>
This standard is more than just slope. It is asking students to find the average rate of change of any function over any given interval. Be sure to include multiple representations (numerically, graphically, or symbolically) of functions for students to work with.

It is an important connection for further courses that students recognize that linear functions have consistent average rate of change over any interval, while functions like quadratics and exponentials do not have constant rates of change due to their curvature.

<table>
<thead>
<tr>
<th>Time (weeks)</th>
<th>Weight (ounces)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>0.04</td>
</tr>
<tr>
<td>9</td>
<td>0.07</td>
</tr>
<tr>
<td>10</td>
<td>0.14</td>
</tr>
<tr>
<td>11</td>
<td>0.25</td>
</tr>
<tr>
<td>12</td>
<td>0.49</td>
</tr>
</tbody>
</table>

(NCDPI Math 1 released EOC #21)

What is the average rate of change in weight of the plankton from week 8 to week 12?

A) 0.0265 ounce per week  
B) 0.0375 ounce per week  
C) 0.055 ounce per week  
D) 0.1125 ounce per week

Example: The table below shows the temperature, T, in Tucson, Arizona t hours after midnight.
When does the temperature decrease the fastest: between midnight and 3 a.m. or between 3 a.m. and 4 a.m.?

<table>
<thead>
<tr>
<th>t (hours after midnight)</th>
<th>0</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>T (temp. in °F)</td>
<td>85</td>
<td>76</td>
<td>70</td>
</tr>
</tbody>
</table>

(https://www.illustrativemathematics.org/content-standards/HSF/IF/B/6/tasks/1500)

Example: You are a marine biologist working for the Environmental Protection Agency (EPA). You are concerned that the rare coral mathemafish population is being threatened by an invasive species known as the fluted dropout shark. The fluted dropout shark is known for decimating whole schools of fish. Using a catch-tag-release method, you collected the following population data over the last year.

<table>
<thead>
<tr>
<th># months since 1st measurement</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathemafish population</td>
<td>480</td>
<td>472</td>
<td>417</td>
<td>318</td>
<td>240</td>
<td>152</td>
<td>103</td>
<td>84</td>
<td>47</td>
<td>32</td>
<td>24</td>
<td>29</td>
<td>46</td>
</tr>
</tbody>
</table>
Through intervention, the EPA was able to reduce the dropout population and slow the decimation of the mathemafish population. Your boss asks you to summarize the effects of the EPA’s intervention plan in order to validate funding for your project.

What to include in your summary report:

a) Calculate the average rate of change of the mathemafish population over specific intervals. Indicate how and why you chose the intervals you chose.

b) When was the population decreasing the fastest?

c) During what month did you notice the largest effects of the EPA intervention?

d) Explain the overall effects of the intervention.

e) Remember to justify all your conclusions using supporting evidence.

(https://www.illustrativemathematics.org/content-standards/HSF/IF/B/6/tasks/686)
**NC.M1.F-LE.1**
*Construct and compare linear and exponential models and solve problems.*

Identify situations that can be modeled with linear and exponential functions, and justify the most appropriate model for a situation based on the rate of change over equal intervals.

<table>
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<tbody>
<tr>
<td><strong>Pre-requisite</strong></td>
</tr>
<tr>
<td>• Construct a function to model a linear relationship (8.F.4)</td>
</tr>
<tr>
<td>• Describe qualitatively the functional relationship between two quantities by analyzing a graph (8.F.5)</td>
</tr>
<tr>
<td>• Formally define a function (NC.M1.F-IF.1)</td>
</tr>
<tr>
<td>• Recognize sequences as function and link arithmetic sequences to linear functions and geometric sequences to exponential functions (NC.M1.F-IF.3)</td>
</tr>
<tr>
<td><strong>Connections</strong></td>
</tr>
<tr>
<td>• Build explicit and recursive forms of arithmetic and geometric sequences (NC.M1.F-BF.1a)</td>
</tr>
<tr>
<td>• Identify key feature of graphs and tables of functions (NC.M1.F-IF.4)</td>
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<tr>
<td>• Identify and interpret key features of functions from different representations (NC.M1.F-IF.7)</td>
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</tr>
<tr>
<td>3 – Construct a viable argument and critique the reasoning of others</td>
</tr>
<tr>
<td>4 – Model with mathematics</td>
</tr>
<tr>
<td>7 – Look for and make use of structure</td>
</tr>
</tbody>
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<tr>
<td><strong>Comprehending the Standard</strong></td>
</tr>
<tr>
<td>Students should differentiate whether a situation (contextual, graphical, or numerical) can be represented best by a linear or exponential model.</td>
</tr>
<tr>
<td>Students should be able to identify whether a situation is linear or exponential based on the context in relation to the rate of change.</td>
</tr>
</tbody>
</table>

| **Assessing for Understanding** |
| Students should be able to identify whether a situation is linear or exponential based on the context and justify their decision. |
| **Example:** Town A adds 10 people per year to its population, and town B grows by 10% each year. In 2006, each town has 145 residents. For each town, determine whether the population growth is linear or exponential. Explain. |

| **Example:** In (a)–(e), say whether the quantity is changing in a linear or exponential fashion. |
| a) A savings account, which earns no interest, receives a deposit of $723 per month. |
| b) The value of a machine depreciates by 17% per year. |
| c) Every week, 9/10 of a radioactive substance remains from the beginning of the week. |

*September 2017 Update - Feedback*
This standard can be taught with NC.MI.F-IF.3 and NC.MI.F-BF.2.

d) A liter of water evaporates from a swimming pool every day.

e) Every 124 minutes, ½ of a drug dosage remains in the body.

(https://www.illustrativemathematics.org/content-standards/HSF/LE/A/1/tasks/629)

<table>
<thead>
<tr>
<th>Time (hours)</th>
<th>Temperature Method 1 (°F)</th>
<th>Temperature Method 2 (°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1.5</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>11</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>19</td>
<td>24</td>
</tr>
<tr>
<td>5</td>
<td>25</td>
<td>48</td>
</tr>
</tbody>
</table>

(NCDPI Math 1 released EOC #24)

Example: Monica did an experiment to compare two methods of warming an object. The results are shown in the table below. Which statement best describes her results?

a) The temperature using both methods changed at a constant rate.
b) The temperature using both methods changed exponentially.
c) The temperature using Method 2 changed at a constant rate.
d) The temperature using Method 2 changed exponentially.

Example: According to Wikipedia, the International Basketball Federation (FIBA) requires that a basketball bounce to a height of 1300 mm when dropped from a height of 1800 mm.

Suppose you drop a basketball and the ratio of each rebound height to the previous rebound height is 1300:1800. Let \( h \) be the function that assigns to \( n \) the rebound height of the ball (in mm) on the \( n \)th bounce.

a) Complete the chart below, rounding to the nearest mm.
b) Write an expression for \( h(n) \).
c) Solve an equation to determine on which bounce the basketball will first have a height of less than 100 mm. (Note: Students are not expected to solve part c algebraically but are expected to take a table or graphical approach.)

(https://www.illustrativemathematics.org/content-standards/HSF/LE/A/1/tasks/347)

Example: For each of the scenarios below, decide whether the situation can be modeled by a linear function, an exponential function, or neither. For those with a linear or exponential model, create a function which accurately describes the situation.

a) From 1910 until 2010 the growth rate of the United States has been steady at about 1.5% per year. The population in 1910 was about 92,000,000.
b) The circumference of a circle as a function of the radius.
c) According to an old legend, an Indian King played a game of chess with a traveling sage on a beautiful, hand-made chessboard. The sage requested, as reward for winning the game, one grain of rice for the first square, two grains for the second, four grains for the third, and so on for the whole chess board. How many grains of rice would the sage win for the \( n \)th square?

(https://www.illustrativemathematics.org/content-standards/HSF/LE/A/1/tasks/629)
NC.M1.F-IF.5
Interpret functions that arise in applications in terms of the context.
Interpret a function in terms of the context by relating its domain and range to its graph and, where applicable, to the quantitative relationship it describes.

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<tr>
<td><strong>Pre-requisite</strong></td>
</tr>
<tr>
<td>● In middle school, students only informally considered restrictions to the domain and range based on context, such as understanding that measurements cannot be negative.</td>
</tr>
<tr>
<td>● Interpret parts of expressions in context (NC.M1.A-SSE.1a, NC.M1.A-SSE.1b)</td>
</tr>
<tr>
<td>● Every point on the graph of an equation is a solution to the equation (NC.M1.A-REI.10)</td>
</tr>
<tr>
<td>● Formally define a function (NC.M1.F-IF.1)</td>
</tr>
<tr>
<td>● Evaluating functions and interpret in context (NC.M1.F-IF.2)</td>
</tr>
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<td><strong>Connections</strong></td>
</tr>
<tr>
<td>● Recognize the domain of sequences (NC.M1.F-IF.3)</td>
</tr>
<tr>
<td>● Identify key feature of graphs and tables of functions (NC.M1.F-IF.4)</td>
</tr>
<tr>
<td>● Analyze linear, quadratic, and exponential functions to identify key features (NC.M1.F-IF.7)</td>
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<td><strong>Comprehending the Standard</strong></td>
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<tr>
<td>Students should be able to associate a reasonable domain and range to a graph as well as to a contextual situation.</td>
</tr>
<tr>
<td>The domain of a graph should be taught in the context of the situation it represents.</td>
</tr>
<tr>
<td><strong>Assessing for Understanding</strong></td>
</tr>
<tr>
<td>Students should be able to identify a reasonable domain and range to its graph as well as to a contextual situation.</td>
</tr>
<tr>
<td><strong>Example:</strong> Jacob is observing bacterial growth in a lab. He noted that the bacteria double in number every hour. There are 50 bacteria at the beginning of his experiment.</td>
</tr>
<tr>
<td>a) Build and graph a function to represent this scenario.</td>
</tr>
<tr>
<td>b) Determine the appropriate domain and range of the function if Jacob runs the experiment for 8 hours.</td>
</tr>
</tbody>
</table>

d) The volume of a cube as a function of its side length.

(stop://www.illustrativemathematics.org/content-standards/HSF/LE/A/1/tasks/1910)
Graphs represented should be both discrete and continuous forms. Students do not need to know the terminology discrete and continuous, but they should be able to identify which is appropriate for each contextual situation.

NC.M1.F-IF.7

*Analyze functions using different representations.*

Analyze linear, exponential, and quadratic functions by generating different representations, by hand in simple cases and using technology for more complicated cases, to show key features, including: domain and range; rate of change; intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximums and minimums; and end behavior.

**Concepts and Skills**

**Pre-requisite**
- Interpret \( y = mx + b \) as being linear (8.F.3)
- Determine rate of change and initial value of linear functions from tables and graphs (8.F.4)
- Interpret parts of expressions in context (NC.M1.A-SSE.1a, NC.M1.A-SSE.1b)
- Formally define a function (NC.M1.F-IF.1)
- Evaluating functions and interpret in context (NC.M1.F-IF.2)
- Identify key feature of graphs and tables of functions (NC.M1.F-IF.4)

**Connections**
- Creating and graphing two variable equations (NC.M1.A-CED.2)
- Solving systems of equations (NC.M1.A-REI.6)
- Recognize the domain of sequences as integers (NC.M1.F-IF.3)
- Relate domain and range of a function to its graph (NC.M1.F-IF.5)
- Calculate the average rate of change (NC.M1.F-IF.6)
- Use equivalent forms of quadratic and exponential function to reveal key features (NC.M1.F-IF.8a, NC.M1.F-IF.8b)
- Compare key features of two functions in different representations (NC.M1.F-IF.9)

**The Standards for Mathematical Practices**

**Implementing the Standards for Mathematical Practice**

**Connections**

*Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.*

4 – Model with mathematics

**Disciplinary Literacy**

*As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.*

Students should be able to justify their use of a representation.

New Vocabulary: exponential function, quadratic function
- Build functions that describe a relationship between two quantities (NC.M1.F-BF.1a, NC.M1.F-BF.1b)
- Identify situations that can be modeled with linear and exponential functions (NC.M1.F-LE.1)
- Interpret the parameters of a linear and exponential function in context (NC.M1.F-LE.5)

<table>
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<tr>
<th>Mastering the Standard</th>
<th>Assessing for Understanding</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Comprehending the Standard</strong></td>
<td>Students should be able to identify key feature of linear, quadratic and exponential functions from the symbolic representation.</td>
</tr>
<tr>
<td>Students should identify the key features of the three function families covered in Math 1: linear, quadratic, and exponential.</td>
<td><strong>Example:</strong> The function $f(x) = 300(0.70)^x - 25$ models the amount of aspirin left in the bloodstream after $y$ hours. Graph the function showing the key features of the graph. Interpret the key features in context of the problem.</td>
</tr>
<tr>
<td>Students should be aware of the key functions typically associated with each function type. Linear functions – domain &amp; range, rate of change, intercepts, increasing/decreasing</td>
<td>Students should be able to identify key feature of exponential functions from the graphical representation.</td>
</tr>
<tr>
<td>Quadratic functions – domain &amp; range, y-intercept, x-intercepts (zeros), intervals of increasing and decreasing, intervals of positive and negative values, maximums and minimums, and end behavior</td>
<td><strong>Example:</strong> Identify the key features of the graph below.</td>
</tr>
<tr>
<td>Exponential functions – domain &amp; range, rate of change, increasing or decreasing (growth and decay), intervals of positive and negative values, and end behavior</td>
<td></td>
</tr>
<tr>
<td>It is important for students to begin developing an understanding of end behavior and interpreting mathematical notation (such as $x \to \infty$). As students study end behavior of these function families, connect their mathematical thinking from “as we keep going out” or “as $x$ gets really big” to “as $x$ goes to infinity”.</td>
<td></td>
</tr>
<tr>
<td>At the Math 1 level, students should <strong>not</strong> be exposed to finding the line of symmetry of a quadratic function using the formula $x = \frac{-b}{2a}$, unless it is developed conceptually. This concept should be developed with a study of the quadratic formula, which will be done in Math 2.</td>
<td></td>
</tr>
</tbody>
</table>
If the students need to find the line of symmetry (not a requirement of Math 1), they can find the midpoint of the zeros of the function.
NC.M1.F-IF.8b

*Analyze functions using different representations.*

Use equivalent expressions to reveal and explain different properties of a function.

b. Interpret and explain growth and decay rates for an exponential function.

### Concepts and Skills

**Pre-requisite**
- Identify and interpret parts of expression (NC.M1.A-SSE.1a, NC.M1.A-SSE.1b)
- Formally define a function (NC.M1.F-IF.1)

**Connections**
- Identify key feature of graphs and tables of functions (NC.M1.F-IF.4)
- Identify and interpret key features of functions from different representations (NC.M1.F-IF.7)
- Compare key features of two functions in different representations (NC.M1.F-IF.9)

### The Standards for Mathematical Practices

**Implementing the Standards for Mathematical Practice**

**Connections**

*Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.*

- 4 – Model with mathematics
- 5 – Use appropriate tools strategically

**Disciplinary Literacy**

*As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.*

New Vocabulary: exponential function, growth rate, decay rate

### Mastering the Standard

**Comprehending the Standard**

This set of standards requires that students rewrite expressions of quadratic and exponential functions to reveal key features of their graphs.

This is the “why” behind rewriting an expression where NC.M1.A-SSE.1 interprets the rate in context. Therefore, these two standards should be taught together.

This standard should also tie to the key features of graphs in NC.M1.F-IF.7

Students should know the key features of an exponential function and how they relate to a contextual situation. Students should be able to find the initial value as well as the growth/decay rate for the interval based on the given

**Assessing for Understanding**

Students should know the key features of an exponential function and how they relate to a contextual situation.

**Example:** The expression $50(0.85)^x$ represents the amount of a drug in milligrams that remains in the bloodstream after $x$ hours.

a) Describe how the amount of drug in milligrams changes over time.

b) What was the initial value of the drug in the bloodstream?

c) What would the expression $50(0.80)^x$ represent?

d) What new or different information is revealed by the changed expression?

**Example:** City Bank pays a simple interest rate of 3% per year, meaning that each year the balance increases by 3% of the initial deposit. National Bank pays an compound interest rate of 2.6% per year, compounded monthly, meaning that each month the balance increases by one twelfth of 2.6% of the previous month’s balance.

a) Which bank will provide the largest balance if you plan to invest $10,000 for 10 years? For 15 years?
b) Write an expression for $C(y)$, the City Bank balance, $y$ years after a deposit is left in the account. Write an expression for $N(m)$, the National Bank balance, $m$ months after a deposit is left in the account.

c) Create a table of values indicating the balances in the two bank accounts from year 1 to year 15. For which years is City Bank a better investment, and for which years is National Bank a better investment?

(https://www.illustrativemathematics.org/content-standards/tasks/302)

NC.M1.F-LE.3

Construct and compare linear and exponential models and solve problems.

Compare the end behavior of linear, exponential, and quadratic functions using graphs and tables to show that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically.

Pre-requisite

- Construct a function to model a linear relationship and interpret rate of change (8.F.4)
- Formally define a function (NC.M1.F-IF.1)
- Evaluate functions (NC.M1.F-IF.2)

Connections

- Calculate the average rate of change of an interval (NC.M1.F-IF.6)
- Identify and interpret key features, like rate of change, of functions from different representations (NC.M1.F-IF.7)

The Standards for Mathematical Practices

Implementing the Standards for Mathematical Practice

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary: exponential function, quadratic function

Mastering the Standard

Comprehending the Standard

Students experiment with the function types to build an understanding that the average rate of change over an interval for an exponential function will eventually surpass the rate of change of a linear function.

Assessing for Understanding

Students should realize that an exponential function is eventually always bigger than a linear or quadratic function.

Example: Kevin and Joseph each decide to invest $100. Kevin decides to invest in an account that will earn $5 every month. Joseph decided to invest in an account that will earn 3% interest every month.

- a) Whose account will have more money in it after two years?
- b) After how many months will the accounts have the same amount of money in them?
- c) Describe what happens as the money is left in the accounts for longer periods of time.
or quadratic function over the same interval. Students should be able to demonstrate this using various representations.

It is important for students to begin developing an understanding of end behavior and interpreting mathematical notation (such as $x \to \infty$). As students study end behavior of these function families, connect their mathematical thinking from “as we keep going out” or “as $x$ gets really big” to “as $x$ goes to infinity”.

**Example:** Using technology, determine the average rate of change of the following functions for intervals of their domains in the table.

<table>
<thead>
<tr>
<th>Functions</th>
<th>Average rate of change</th>
<th>Average rate of change</th>
<th>Average rate of change</th>
<th>Average rate of change</th>
<th>Average rate of change</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x) = x^2$</td>
<td>$0 \leq x \leq 10$</td>
<td>$10 \leq x \leq 20$</td>
<td>$20 \leq x \leq 30$</td>
<td>$30 \leq x \leq 40$</td>
<td>$40 \leq x \leq 50$</td>
</tr>
<tr>
<td>$f(x) = 1.17^x$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a) When does the average rate of change of the exponential function exceed the average rate of change of the quadratic function?
b) Using a graphing technology, graph both functions. How do the average rates of change relate to what you see on the graph?  
   Note: You can use the information in your table to determine how to change the setting to see where the functions intersect.
c) In your graphing technology, change the first function to $f(x) = 10x^2$ and adjust the settings to see where the functions intersect. What do you notice about the rates of change interpreted from the graph?
d) Make a hypothesis about the rates of change about polynomial and exponential function. Try other values for the coefficient of the quadratic function to support your hypothesis.

**NC.M1.A-SSE.1a**

*Interpret the structure of expressions.*

Interpret expressions that represent a quantity in terms of its context.

- Identify and interpret parts of a linear, exponential, or quadratic expression, including terms, factors, coefficients, and exponents.
## Connections

- Interpreting changes in the parameters of a linear and exponential function in context (NC.M1.F-LE.5)

## Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.
- New Vocabulary: Quadratic term, exponential term

## Mastering the Standard

### Comprehending the Standard

The set of A-SSE standards requires students:
- to write expressions in equivalent forms to reveal key quantities in terms of its context.
- to choose and use appropriate mathematics to analyze situations.

For this part of the standards, students recognize that the linear expression \( mx + b \) has two terms, that \( m \) is a coefficient, and \( b \) is a constant.

Students are expected to recognize the parts of a quadratic expression, such as the quadratic, linear and constant term, or factors.

For exponential expressions, students should recognize factors, the base, and exponent(s).

Students extend beyond simplifying to interpret the components of an algebraic expression.

### Assessing for Understanding

Students should recognize that in the expression \( 2x + 1 \), “2” is the coefficient, “2” and “x” are factors, and “1” is a constant, as well as “2x” and “1” being terms of the binomial expression. Also, a student recognizes that in the expression \( 4(3)^x \), 4 is the coefficient, 3 is the factor, and \( x \) is the exponent. Development and proper use of mathematical language is an important building block for future content. Using real-world context examples, the nature of algebraic expressions can be explored.

**Example:** The height (in feet) of a balloon filled with helium can be expressed by \( 5 + 6.3s \) where \( s \) is the number of seconds since the balloon was released. Identify and interpret the terms and coefficients of the expression.

**Example:** The expression \(-4.9t^2 + 17t + 0.6\) describes the height in meters of a basketball \( t \) seconds after it has been thrown vertically into the air. Interpret the terms and coefficients of the expression in the context of this situation.

**Example:** The expression \(35000(0.87)^t\) describes the cost of a new car \( t \) years after it has been purchased. Interpret the terms and coefficients of the expression in the context of this situation.
NC.M1.F-IF.9

**Analyze functions using different representations.**

Compare key features of two functions (linear, quadratic, or exponential) each with a different representation (symbolically, graphically, numerically in tables, or by verbal descriptions).

## Concepts and Skills

### Pre-requisite

- Compare properties of two functions each represented in different ways (8.F.2)
- Formally define a function (NC.M1.F-IF.1)
- Identify key feature of graphs and tables of functions (NC.M1.F-IF.4)
- Identify and interpret key features of functions from different representations (NC.M1.F-IF.7)
- Rewrite quadratic functions to identify key features (NC.M1.F-IF.8a)
- Interpret and explain growth and decay rates for an exponential function (NC.M1.F-IF.8b)

### Connections

**The Standards for Mathematical Practices**

**Implementing the Standards for Mathematical Practice**

- **Connections**
  - Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.
  - 4 – Model with mathematics
  - 5 – Use appropriate tools strategically

**Disciplinary Literacy**

- As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.
- Students should be able to justify their use of a representation to make the comparison.
- New Vocabulary: exponential function, quadratic function

## Mastering the Standard

### Comprehending the Standard

Students should compare two functions in two different forms. The function types may be the same (linear & linear) or different (linear & exponential), but the representations should be different (e.g. numerical & graphical).

It is important to note that the point of this standard is not to have students simply translate one function into

### Assessing for Understanding

**Example:** Suppose Brett and Andre each throws a baseball into the air. The height of Brett's baseball is given by \( h(t) = -16t^2 + 79t + 6 \), where \( h \) is in feet and \( t \) is in seconds. The height of Andre's baseball is given by the graph below:
the same form as the other function when given in different forms. Students should be able to use appropriate tools to compare the key features of functions.

Brett claims that his baseball went higher than Andre's, and Andre says that his baseball went higher.

a) Who is right?
b) How long is each baseball airborne?
c) Construct a graph of the height of Brett's throw as a function of time on the same set of axes as the graph of Andre's throw (if not done already), and explain how this can confirm your claims to parts (a) and (b).

Example: Dennis compared the y-intercept of the graph of the function $f(x) = 3x + 5$ to the y-intercept of the graph of the linear function that includes the points in the table below.

<table>
<thead>
<tr>
<th>x</th>
<th>g(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>−7</td>
<td>2</td>
</tr>
<tr>
<td>−5</td>
<td>3</td>
</tr>
<tr>
<td>−3</td>
<td>4</td>
</tr>
<tr>
<td>−1</td>
<td>5</td>
</tr>
</tbody>
</table>

What is the difference when the y-intercept of $f(f)$ is subtracted from the y-intercept of $g(x)$?

A. $−11.0$
B. $−9.3$
C. $0.5$
D. $5.5$

Example: Joe is trying to decide which job would allow him to earn the most money after a few years.

- His first job offer agrees to pay him $500 per week. If he does a good job, they will give him a 2% raise each year.
- His other job offer agrees to pay him according to the following equation $f(x) = 20,800(1.03)^x$, where $x$ represents the number of years and $f(x)$ his salary.

Which job would you suggest Joe take? Justify your reasoning.

Example: Mario compared the slope of the function graphed below to the slope of the linear function that has an $x$-intercept of $\frac{4}{3}$ and a $y$-intercept of $−2$. 
What is the slope of the function with the smaller slope?
A) $\frac{1}{5}$
B) $\frac{1}{3}$
C) $\frac{3}{5}$
D) $\frac{5}{3}$
(NCDPI Math 1 EOC released #25)
NC.M1.A-REI.11

**Represent and solve equations and inequalities graphically**

Build an understanding of why the $x$-coordinates of the points where the graphs of two linear, exponential, or quadratic equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$ and approximate solutions using a graphing technology or successive approximations with a table of values.

### Concepts and Skills

**Pre-requisite**
- Solving multi-step linear equations (8.EE.7)
- Analyze and solve pairs of simultaneous linear equations by graphing and substitution (8.EE.8)
- Understand every point on a graph is a solution to its associated equation (NC.M1.A-REI.10)

**Connections**
- Creating and solving one variable equations and systems of equations (NC.M1.A-CED.1, NC.M1.A-CED.3)
- Solving systems of equations (NC.M1.A-REI.6)

**The Standards for Mathematical Practices**

**Implementing the Standards for Mathematical Practice**

**Connections**

*Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.*

4 – Model with mathematics
6 – Attend to precision

**Disciplinary Literacy**

*As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.*

New Vocabulary: exponential function, quadratic function

### Mastering the Standard

**Comprehending the Standard**

For a complete understanding, students will need exposure to both parts of this standard. Students should be able to see the connection between graphs and tables of two functions, the points they have in common and the truthfulness of the equation. Because $f(x) = g(x)$ when $x = 3$, 3 is the solution to the equation $2x - 4 = \frac{1}{2}x + \frac{1}{2}$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x) = 2x - 4$</th>
<th>$x$</th>
<th>$g(x) = \frac{1}{2}x + \frac{1}{2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-4</td>
<td>0</td>
<td>.5</td>
</tr>
<tr>
<td>1</td>
<td>-2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>2</td>
<td>1.5</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>4</td>
<td>2.5</td>
</tr>
</tbody>
</table>

In Math 1, students are expected to solve linear equations using inverse operations and quadratic equations with square roots and factoring. In all other equations, such as exponential equations, solutions should be approximated with technology, tables and graphs.

### Assessing for Understanding

Students should be able to find approximate solutions to exponential equations using technology, tables and graphs.

**Example:** Solve the following equations by graphing. Give your answer to the nearest tenth.

a) $3(2^x) = 6x - 7$

b) $10x + 5 = -x + 8$

**Example:** The population of a country is initially 2 million people and is increasing at 4% per year. The country’s annual food supply is initially adequate for 4 million people and is increasing at a constant rate adequate for an additional 0.5 million people per year.

a) Based on these assumptions, in approximately what year will this country first experience shortages of food?
For example:

Solve: \(3x^2 - 2x + 1 = \frac{1}{2}x + 5\)

Rewrite the equations as a system of equations:

\[f(x) = 3x^2 - 2x + 1\]
\[g(x) = \frac{1}{2}x + 5\]

Using technology, graph the equations and look for points of intersection, where the same \(x\) produces \(f(x) = g(x)\).

b) If the country doubled its initial food supply and maintained a constant rate of increase in the supply adequate for an additional 0.5 million people per year, would shortages still occur? In approximately which year?

c) If the country doubled the rate at which its food supply increases, in addition to doubling its initial food supply, would shortages still occur?

(https://www.illustrativemathematics.org/content-standards/HSA/REI/D/11/tasks/645)

**NC.M1.F-BF.1a**

*Build a function that models a relationship between two quantities.*

Write a function that describes a relationship between two quantities.

a. Build linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two ordered pairs (include reading these from a table).
Comprehending the Standard

This standard is about building a function from different representations. In this part of the standard, the different representations include: sequences, graphs, verbal descriptions, tables, and ordered pairs.

This standard pairs well with Interpreting Functions standards, in that the purpose behind building a function is to then use that function to solve a problem.

These functions can be written in function notation (linear or exponential) or as a sequence in explicit or recursive form. Students should recognize explicit form of an arithmetic sequence as an equivalent structure to slope-intercept form of a linear function and explicit form of a geometric sequence as an equivalent structure to standard form of an exponential function. Using the concepts of rate of change, students should recognize that the forms of these sequences are one iteration forward from the y-intercept, which gives meaning to the \( n - 1 \) notation.

Assessing for Understanding

Students should write functions from verbal descriptions as well as a table of values.

**Example:** Suppose a single bacterium lands on one of your teeth and starts reproducing by a factor of 2 every hour. If nothing is done to stop the growth of the bacteria, write a function for the number of bacteria as a function of the number of days.

**Example:** The table below shows the cost of a pizza based on the number of toppings.

<table>
<thead>
<tr>
<th>Number of Toppings</th>
<th>Cost (C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$12.00</td>
</tr>
<tr>
<td>2</td>
<td>$13.50</td>
</tr>
<tr>
<td>3</td>
<td>$15.00</td>
</tr>
<tr>
<td>4</td>
<td>$16.50</td>
</tr>
</tbody>
</table>

Which function represents the cost of a pizza with \( n \) toppings?
- A) \( C(n) = 12 + 1.5(n - 1) \)
- B) \( C(n) = 1.5n + 12 \)
- C) \( C(n) = 12 + n \)
- D) \( C(n) = 12n \)

(NCDPI Math 1 released EOC #39)

**Example:** The height of a stack of cups is a function of the number of cups in the stack. If a 7.5” cup with a 1.5” lip is stacked vertically, determine a function that would provide you with the height based on any number of cups.

*Hint:* Start with height of one cup and create a table, list, graph or description that describes the pattern of the stack as an additional cup is added.

**Example:** There were originally 4 trees in an orchard. Each year the owner planted the same number of trees. In the 29th year, there were 178 trees in the orchard. Which function, \( t(n) \), can be used to determine the number of trees in the orchard in any year, \( n \)?
- A) \( t(n) = \frac{178}{29} n + 4 \)
- B) \( t(n) = \frac{178}{29} n - 4 \)
- C) \( t(n) = 6n + 4 \)
- D) \( t(n) = 29n - 4 \)

(NCDPI Math 1 released EOC #42)

Students should write linear or exponential relationships as a sequence in explicit or recursive form.
Example: The price of a new computer decreases with age. Examine the table by analyzing the outputs.
   a) Describe the recursive relationship.
   b) Analyze the input and the output pairs to determine an explicit function that represents the value of the computer when the age is known.

<table>
<thead>
<tr>
<th>Age</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$1575</td>
</tr>
<tr>
<td>2</td>
<td>$1200</td>
</tr>
<tr>
<td>3</td>
<td>$900</td>
</tr>
<tr>
<td>4</td>
<td>$650</td>
</tr>
<tr>
<td>5</td>
<td>$500</td>
</tr>
<tr>
<td>6</td>
<td>$400</td>
</tr>
<tr>
<td>7</td>
<td>$300</td>
</tr>
</tbody>
</table>

NC.M1.F-BF.1b

*Build a function that models a relationship between two quantities.*
Write a function that describes a relationship between two quantities.
   b. Build a function that models a relationship between two quantities by combining linear, exponential, or quadratic functions with addition and subtraction or two linear functions with multiplication.
this part of the standard students should combine functions to represent a contextual situation.

This standard pairs well with Interpreting Functions standards, in that the purpose behind building a function is to then use that function to solve a problem.

The algebraic skills behind this standard occur in NC.M1.A-APR.1. This standard should be taught throughout the year as each new function family is added to the course.

---

Example: Cell phone Company Y charges a $10 start-up fee plus $0.10 per minute, \( x \). Cell phone Company Z charges $0.20 per minute, \( x \), with no start-up fee. Which function represents the difference in cost between Company Y and Company Z?

A) \( f(x) = -0.10x - 10 \)
B) \( f(x) = -0.10x + 10 \)
C) \( f(x) = 10x - 0.10 \)
D) \( f(x) = 10x + 0.10 \)

(NCDPI Math 1 released EOC #23)

Example: A retail store has two options for discounting items to go on clearance.

- Option 1: Decrease the price of the item by 15% each week.
- Option 2: Decrease the price of the item by $5 each week.

If the cost of an item is $45, write a function rule for the difference in price between the two options.

Example: Blake has a monthly car payment of $225. He has estimated an average cost of $0.32 per mile for gas and maintenance. He plans to budget for the car payment the minimal he needs with an additional 3% of his total budget for incidentals that may occur. Build a function that gives the amount Blake needs to budget as a function of the number of miles driven.

---

NC.M1.S-ID.6c

Summarize, represent, and interpret data on two categorical and quantitative variables.

Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

- Fit a function to exponential data using technology. Use the fitted function to solve problems.

---

<table>
<thead>
<tr>
<th>Concepts and Skills</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pre-requisite</strong></td>
</tr>
<tr>
<td>Fit a regression line to linear data using technology (NC.M1.S-ID.6a)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>The Standards for Mathematical Practices</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Implementing the Standards for Mathematical Practice</strong></td>
</tr>
<tr>
<td><strong>Connections</strong></td>
</tr>
<tr>
<td>Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.</td>
</tr>
<tr>
<td>4 – Model with mathematics</td>
</tr>
<tr>
<td>5 – Use appropriate tools strategically</td>
</tr>
<tr>
<td>6 – Attend to precision</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Vocabulary</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in</td>
</tr>
</tbody>
</table>

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September 2017 Update  -  Feedback  NC Math 1 Unit 3 Introduction to Exponential Functions  Return to: Standards Addressed in this Unit
(NC.M1.A-CED.1)
- Recognize a geometric sequence as a subset of the range of an exponential function (NC.M1.F-IF.3)
- Exponential growth and decay (NC.M1.F-IF.8b)
- Use technology to analyze patterns and describe relationships between two variables in context. (NC.M1.S-ID.7)
- Identify situations that can be modeled with linear and exponential functions, and justify the most appropriate model (NC.M1.F-LE.1)
- Interpret the parameters in linear or exponential functions in terms of a context (NC.M1.F-LE.5)
- Interpret key features in context to describe functions relating two quantities (NC.M1.F-IF.4)
- Interpret a function in terms of its domain and range in context (NC.M1.F-IF.5)
- Calculate and interpret the avg. rate of change for a function (NC.M1.F-IF.6)

all oral and written communication. The following vocabulary is new to this course and supported by this standard:

<table>
<thead>
<tr>
<th>Mastering the Standard</th>
<th>Assessing for Understanding</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comprehending the Standard</td>
<td>Students can use graphing technology or a graphing calculator to determine the exponential model for a given data set or scatter plot.</td>
</tr>
<tr>
<td>Work with exponential functions is new to students. In 8th grade, students focused on identifying characteristics of linear functions and distinguishing them from non-linear functions. Students will use the same tools to explore exponential functions specifically. This standard should be explored in context to help students make meaning of the behavior of exponential models. Technology can be used as a tool to make connections between symbolic, tabular and graphical representations of exponential functions. This will also help to build conceptual understanding of exponential growth and decay. At this level, students should be able to support the use of an exponential model based on the graphical display and the</td>
<td></td>
</tr>
<tr>
<td>Students can make connections between the graph, table, and symbolic representations of an exponential function.</td>
<td></td>
</tr>
<tr>
<td>Example: In an experiment, 300 pennies were shaken in a cup and poured onto a table. Any penny ‘heads up’ was removed. The remaining pennies were returned to the cup and the process was repeated. The results of the experiment are shown below. Write a function rule suggested by the context. Use the context to explain all values of the function. How are those values reflected in the table?</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Hours</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of gnats</td>
<td>12</td>
<td>20</td>
<td>35</td>
<td>60</td>
<td>80</td>
</tr>
</tbody>
</table>
Students should be presented with exponential data and asked to fit the function to the data using technology. They should NOT have to verify the appropriateness of an exponential model; analysis at that level requires transformations for linearity, which is an advanced statistical concept.

<table>
<thead>
<tr>
<th># of Rolls</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td># of Pennies</td>
<td>300</td>
<td>164</td>
<td>100</td>
<td>46</td>
<td>20</td>
<td>8</td>
</tr>
</tbody>
</table>
RESEARCH BRIEF: Quadratic Functions

Essential Questions:
- What patterns of change are involved in projectile motion?
- What patterns of change appear in tables and graphs of values for quadratic functions?
- What functions model patterns of change that appear in tables and graphs of values for projectile motion?
- How can tables, graphs, and equations for quadratic functions be used to answer questions about the situations they represent?
- How are the values of $a$, $b$, and $c$ related to patterns in the graphs and tables of values for quadratic functions in the form $f(x) = ax^2+bx+c$?
- How can you predict the shape and location of graphs of quadratic functions with rules in the form $y=ax^2$?
- How can you predict the shape and location of graphs of quadratic functions with rules in the form $y=ax^2+c$?
- How can you predict the shape and location of graphs of quadratic functions with rules in the form $y=ax^2+bx$?
- How can you predict the shape and location of graphs of quadratic functions with rules in the form $y=ax^2+bx+c$?
- What strategies are useful in finding rules for quadratic functions?
- How are quadratic functions used in the real world?
- What strategies are useful in deciding whether two quadratic expressions are equivalent?
- What strategies are useful in deciding when one form of quadratic expression is more useful than another?
- What strategies can be used to transform quadratic expressions into useful equivalent forms?
- What are some effective methods for solving quadratic expressions algebraically?
- How do you determine when one method of solving quadratic is more efficient than another?
<table>
<thead>
<tr>
<th>Learning Outcomes</th>
<th>Student Objectives</th>
</tr>
</thead>
<tbody>
<tr>
<td>● Students will use all arithmetic operations on polynomials.</td>
<td>● I will <strong>describe</strong> the patterns of change in quadratic functions.</td>
</tr>
<tr>
<td>● Students will determine patterns of change associated with quadratic functions.</td>
<td>● I will <strong>evaluate</strong> tables and graphs to predict solutions of a quadratic model in context.</td>
</tr>
<tr>
<td>● Students will use tables of values and graphs to estimate answers for questions about situations modeled by quadratic functions.</td>
<td>● I will <strong>describe</strong> how changing each part (a, b, c) of a quadratic equation alters the function.</td>
</tr>
<tr>
<td>● Students will describe the effects of each parameter in the function rule (y=ax^2+bx+c)</td>
<td>● I will <strong>calculate</strong> the quadratic regression given a set of data.</td>
</tr>
<tr>
<td>● Students will determine whether two quadratic expressions are equivalent.</td>
<td>● I will <strong>justify</strong> whether a pair of quadratic equations are equivalent.</td>
</tr>
<tr>
<td>● Students will determine the most useful form of a quadratic function depending on s for different question types.</td>
<td>● I will <strong>determine</strong> which quadratic form is best in the context of the problem.</td>
</tr>
<tr>
<td>● Students will create equivalent quadratic expressions by expanding products of linear factors using the distributive property.</td>
<td>● I will <strong>use</strong> the roots and/or factors of a polynomial to <strong>write</strong> a quadratic in standard form.</td>
</tr>
<tr>
<td>● Students will factor and solve quadratic equations using the greatest common factor.</td>
<td>● I will <strong>solve</strong> a quadratic equation by factoring (GCF) or extracting the roots.</td>
</tr>
<tr>
<td>● Students will solve quadratic equations using square roots.</td>
<td>● I will <strong>solve</strong> a quadratic equation by factoring (binomial factors - difference of square, trinomials with and without leading coefficient).</td>
</tr>
<tr>
<td>● Students will factor quadratic equations into binomial factors and solve using the zero-product property.</td>
<td>● I will <strong>create</strong> a quadratic equation or inequality from a given scenario.</td>
</tr>
<tr>
<td>● Students will write quadratic equations and inequalities to express questions about quadratic functions.</td>
<td>● I will <strong>understand</strong> the relationship between factors and roots, solutions, x-intercepts, and zeros.</td>
</tr>
<tr>
<td>● Students will understand the relationship between factors and roots, solutions, x-intercepts, and zeros of a quadratic function.</td>
<td>● I will <strong>locate</strong> and <strong>explain</strong> the vertex, x-intercept, and y-intercept of a quadratic given a graph, an equation, and in context of a situation.</td>
</tr>
<tr>
<td>● Students will solve a system of equations of a linear function and a quadratic function.</td>
<td>● I will <strong>describe</strong> key features of quadratic functions in context of the scenario they represent.</td>
</tr>
</tbody>
</table>
- Students will determine the vertex, x-intercept, and y-intercept from a graph, equation and in context with and without a graphing calculator.

### Standards Addressed in this Unit

**Understand the terms and properties of polynomials.**

<table>
<thead>
<tr>
<th>OCS Priority Standard(s)</th>
<th>Supporting Standard(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>NC.M1.A-SSE.1a:</strong> Interpret the structure of expressions. Interpret expressions that represent a quantity in terms of its context. Identify and interpret parts of a linear, exponential, or quadratic expression, including terms, factors, coefficients, and exponents.</td>
<td><strong>NC.M1.A-APR.1:</strong> Perform arithmetic operations on polynomials. Build an understanding that operations with polynomials are comparable to operations with integers by adding and subtracting quadratic expressions and by adding, subtracting, and multiplying linear expressions.</td>
</tr>
<tr>
<td><strong>NC.M1.A-SSE.1b:</strong> Interpret the structure of expressions. Interpret expressions that represent a quantity in terms of its context. Interpret a linear, exponential, or quadratic expression made of multiple parts as a combination of entities to give meaning to an expression.</td>
<td></td>
</tr>
</tbody>
</table>

**Understand how changing the coefficients of a quadratic expression affect the key features of its related quadratic function.**

<table>
<thead>
<tr>
<th>OCS Priority Standard(s)</th>
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</thead>
<tbody>
<tr>
<td><strong>NC.M1.F-IF.7:</strong> Analyze functions using different representations. Analyze linear, exponential, and quadratic</td>
<td><strong>NC.M1.F-LE.3:</strong> Construct and compare linear, exponential, and quadratic models and solve problems. Compare the end behavior of linear,</td>
</tr>
</tbody>
</table>

[Return to Standards Addressed in this Unit]
functions by generating different representations, by hand in simple cases and using technology for more complicated cases, to show key features, including: domain and range; rate of change; intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximums and minimums; and end behavior. **NC.M1.F-IF.9:** Analyze functions using different representations. Compare key features of two functions (linear, quadratic, or exponential) each with a different representation (symbolically, graphically, numerically in tables, or by verbal descriptions). **NC.M1.A-REI.10:** Represent and solve equations and inequalities graphically. Understand that the graph of a two variable equation represents the set of all solutions to the equation. **NC.M1.A-CED.1:** Create equations and inequalities in one variable that represent linear, exponential and quadratic relationships and use them to solve problems. **NC.M1.A-CED.2:** Create and graph equations in two variables to represent linear exponential, and quadratic relationships between quantities.

exponential, and quadratic functions using graphs and tables to show that a quantity increasing exponentially exceeds a quantity increasing linearly or quadratically. **NC.M1.F-IF.6:** Interpret functions that arise in applications in terms of the context. Calculate and interpret the average rate of change over a specified interval for a function presented numerically, graphically, and/or symbolically. **NC.M1.F-IF.8a:** Analyze functions using different representations. Use equivalent expressions to reveal and explain different properties of a function. Rewrite a quadratic function to reveal and explain different key features of the function. **NC.M1.F-IF2:** Understand the concept of a function and use function notation. Use function notation to evaluate linear, quadratic, and exponential functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

<table>
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<tbody>
<tr>
<td><strong>NC.M1.A-SSE.3:</strong> Write an equivalent form of a quadratic expression by factoring, where $a$ is an integer of the quadratic expression, $ax^2 + bx + c$, and $c$ is an integer.</td>
<td><strong>NC.M1.A-CED.2:</strong> Create and graph equations in two variables to represent linear exponential, and quadratic relationships between quantities.</td>
</tr>
</tbody>
</table>
c, to reveal the solutions of the equation or the zeros of the function the expression defines.

**NC.M1.A.REI.4:** Solve for the real solutions of quadratic equations in one variable by taking square roots and factoring.

**NC.M1.A-APR.3:** Understand the relationship between zeros and factors of polynomials. Understand the relationships among the factors of a quadratic expression, the solutions of a quadratic equation, and the zeros of a quadratic function.

**NC.M1.A-REI.1:** Understand solving equations as a process of reasoning and explain the reasoning. Justify a chosen solution method and each step of the solving process for linear and quadratic equations using mathematical reasoning.

**NC.M1.A-REI.11:** Build an understanding of why the x-coordinates of the points where the graphs of two linear, exponential, or quadratic equations, \( y = f(x) \) and \( y = g(x) \), intersect are the solutions of the equation \( f(x) = g(x) \) and approximate solutions using a graphing technology or successive approximations with a table of values.

---

**Build quadratic functions from other functions and interpret the key features of a quadratic function in context.**

<table>
<thead>
<tr>
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<tbody>
<tr>
<td><strong>NC.M1.F-IF.4:</strong> Interpret functions that arise in applications in terms of the context. Interpret key features of graphs, tables, and verbal descriptions in context to describe functions that arise in applications relating two quantities, including: intercepts; intervals where the function is increasing, decreasing, positive, or negative; and maximums and minimums.</td>
<td><strong>NC.M1.A-APR.1:</strong> Perform arithmetic operations on polynomials. Build an understanding that operations with polynomials are comparable to operations with integers by adding and subtracting quadratic expressions and by adding, subtracting, and multiplying linear expressions. <strong>NC.M1.F-BF.1b:</strong> Build a function that models a relationship between two quantities.</td>
</tr>
<tr>
<td>Standards Addressed in this Unit</td>
<td></td>
</tr>
<tr>
<td>----------------------------------</td>
<td></td>
</tr>
</tbody>
</table>
| **NC.M1.A-SSE.1b:** Interpret the structure of expressions. Interpret expressions that represent a quantity in terms of its context. Interpret a linear, exponential, or quadratic expression made of multiple parts as a combination of entities to give meaning to an expression.  
**NC.M1.F-IF.7:** Analyze functions using different representations. Analyze linear, exponential, and quadratic functions by generating different representations, by hand in simple cases and using technology for more complicated cases, to show key features, including: domain and range; rate of change; intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximums and minimums; and end behavior.  
**NC.M1.F-IF.5:** Interpret a function in terms of context by relating its domain and range to its graph and, where applicable, to the quantitative relationship it describes.  
**NC.M1.A-REI.11:** Build an understanding of why the x-coordinates of the points where the graphs of two linear, exponential, or quadratic equations \( y = f(x) \) and \( y = g(x) \) intersect are the solutions of the equation \( f(x) = g(x) \) and approximate solutions using a graphing technology or successive approximations with a table of values.  
**NC.M1.S-ID.8:** Interpret linear models. Analyze patterns and describe relationships between two variables in context. Using technology, determine the correlation coefficient of bivariate data and interpret it as a measure of the strength and direction of a linear relationship. Use a scatter plot, correlation coefficient, and a residual plot to determine the appropriateness of using a linear function to model a relationship between two variables.  
|
**NC.M1.A-APR.1**  
*Perform arithmetic operations on polynomials.*  
Build an understanding that operations with polynomials are comparable to operations with integers by adding and subtracting quadratic expressions and by adding, subtracting, and multiplying linear expressions.

### Concepts and Skills

<table>
<thead>
<tr>
<th>Pre-requisite</th>
</tr>
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<tbody>
<tr>
<td>● Add, subtract, factor and expand linear expressions (7.EE.1)</td>
</tr>
<tr>
<td>● Understand that rewriting expressions into equivalent forms can reveal other relationships between quantities (7.EE.2)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Connections</th>
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<tbody>
<tr>
<td>● Rewrite expressions using the properties of exponents (NC.M1.N-RN.2)</td>
</tr>
<tr>
<td>● Understanding the process of elimination (NC.M1.A-REI.5)</td>
</tr>
<tr>
<td>● Rewrite a quadratic function to reveal key features (NC.M1.F-IF.8a)</td>
</tr>
<tr>
<td>● Building functions to model a relationship (NC.M1.F-BF.1b)</td>
</tr>
</tbody>
</table>

### The Standards for Mathematical Practices

<table>
<thead>
<tr>
<th>Implementing the Standards for Mathematical Practice</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Connections</strong></td>
</tr>
<tr>
<td>Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.</td>
</tr>
<tr>
<td>2 – Reason abstractly and quantitatively</td>
</tr>
<tr>
<td>7 – Look for and make use of structure</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Disciplinary Literacy</th>
</tr>
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<tbody>
<tr>
<td>As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.</td>
</tr>
</tbody>
</table>

Students should be able to compare operations with polynomials to operations with integers.

New Vocabulary: polynomial, quadratic expression

### Mastering the Standard

<table>
<thead>
<tr>
<th>Comprehending the Standard</th>
<th>Assessing for Understanding</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students connect their knowledge of integer operations to polynomial operations.</td>
<td>Students should be able to rewrite polynomial expressions using the properties of operations.</td>
</tr>
</tbody>
</table>
| At the Math 1 level, students are only responsible for the following operations:  
  ● adding and subtracting quadratic expressions  
  ● adding, subtracting, and multiplying linear expressions | **Example:** Write at least two equivalent expressions for the area of the circle with a radius of \( 5x - 2 \) kilometers. |
|  | **Example:** Simplify each of the following:  
  a) \( (4x + 3) - (2x + 1) \)  
  b) \( (x^2 + 5x - 9) + 2x(4x - 3) \)  
  | **Example:** The area of a trapezoid is found using the formula \( A = \frac{1}{2} h(b_1 + b_2) \), where \( A \) is the area, \( h \) is the height, and \( b_1 \) and \( b_2 \) are the lengths of the bases. |
What is the area of the trapezoid?
A) \( A = 4x + 2 \)
B) \( A = 4x + 8 \)
C) \( A = 2x^2 + 4x - 21 \)
D) \( A = 2x^2 + 8x - 42 \)

(NCDPI Math I released EOC #33)

**Example:** A town council plans to build a public parking lot. The outline below represents the proposed shape of the parking lot.

a) Write an expression for the area, in square feet, of this proposed parking lot. Explain the reasoning you used to find the expression.

b) The town council has plans to double the area of the parking lot in a few years. They plan to increase the length of the base of the parking lot by \( p \) yards, as shown in the diagram below.

c) Write an expression in terms of \( x \) to represent the value of \( p \), in feet. Explain the reasoning you used to find the value of \( p \).

**Example:** A cardboard box as a height of \( x \), a width that is 3 units longer than the height, and a length that is 2 units longer than the width. Write an expression in terms of \( x \) to represent the volume of the box.
NC.M1.A-SSE.1a

*Interpret the structure of expressions.*

Interpret expressions that represent a quantity in terms of its context.

- Identify and interpret parts of a linear, exponential, or quadratic expression, including terms, factors, coefficients, and exponents.

### Concepts and Skills

**Pre-requisite**

- Identify parts of an expression using precise vocabulary (6.EE.2b)
- Interpret numerical expressions written in scientific notation (8.EE.4)
- For linear and constant terms in functions, interpret the rate of change and the initial value (8.F.4)

**Connections**

- Interpreting changes in the parameters of a linear and exponential function in context (NC.M1.F-LE.5)

**The Standards for Mathematical Practices**

**Implementing the Standards for Mathematical Practice**

**Connections**

*Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.*

2 – Reason abstractly and quantitatively.

4 – Model with mathematics

7 – Look for and make use of structure.

**Disciplinary Literacy**

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary: Quadratic term, exponential term

### Mastering the Standard

**Comprehending the Standard**

The set of A-SSE standards requires students:

- to write expressions in equivalent forms to reveal key quantities in terms of its context.
- to choose and use appropriate mathematics to analyze situations.

For this part of the standards, students recognize that the linear expression \( mx + b \) has two terms, that \( m \) is a coefficient, and \( b \) is a constant.

Students are expected to recognize the parts of a quadratic expression, such as the quadratic, linear and constant term, or factors.

For exponential expressions, students should recognize

**Assessing for Understanding**

Students should recognize that in the expression \( 2x + 1 \), “2” is the coefficient, “2” and “x” are factors, and “1” is a constant, as well as “2x” and “1” being terms of the binomial expression. Also, a student recognizes that in the expression \( 4(3)^x \), 4 is the coefficient, 3 is the factor, and \( x \) is the exponent. Development and proper use of mathematical language is an important building block for future content. Using real-world context examples, the nature of algebraic expressions can be explored.

**Example:** The expression \(-4.9t^2 + 17t + 0.6\) describes the height in meters of a basketball \( t \) seconds after it has been thrown vertically into the air. Interpret the terms and coefficients of the expression in the context of this situation.
NC.M1.A-SSE.1b

Interpret the structure of expressions.

Interpret expressons that represent a quantity in terms of its context.

b. Interpret a linear, exponential, or quadratic expression made of multiple parts as a combination of entities to give meaning to an expression.

---

The Standards for Mathematical Practices

Implementing the Standards for Mathematical Practice

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively.
4 – Model with mathematics
7 – Look for and make use of structure.

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary: exponential expression, quadratic expression

---

Mastering the Standard

Comprehending the Standard

The set of A-SSE standards requires students:

- to write expressions in equivalent forms to reveal key quantities in terms of its

Assessing for Understanding

Students should understand that working with unsimplified expressions often reveals key information from a context.

**Example:** A rectangle has a length that is 2 units longer than the width. If the width is increased by 4 units and the length increased by 3 units, write two equivalent expressions for the area of the rectangle.
NC.M1.F-IF.7

Analyze functions using different representations.

Analyze linear, exponential, and quadratic functions by generating different representations, by hand in simple cases and using technology for more complicated cases, to show key features, including: domain and range; rate of change; intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximums and minimums; and end behavior.

<table>
<thead>
<tr>
<th>Concepts and Skills</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pre-requisite</strong></td>
</tr>
<tr>
<td>● Interpret ( y = mx + b ) as being linear (8.F.3)</td>
</tr>
<tr>
<td>● Determine rate of change and initial value of linear functions from tables and graphs (8.F.4)</td>
</tr>
<tr>
<td>● Interpret parts of expressions in context (NC.M1.A-SSE.1a, NC.M1.A-SSE.1b)</td>
</tr>
<tr>
<td>● Formally define a function (NC.M1.F-IF.1)</td>
</tr>
<tr>
<td>● Evaluating functions and interpret in context (NC.M1.F-IF.2)</td>
</tr>
<tr>
<td>● Identify key feature of graphs and tables of functions (NC.M1.F-IF.4)</td>
</tr>
<tr>
<td><strong>Connections</strong></td>
</tr>
<tr>
<td>● Creating and graphing two variable equations (NC.M1.A-CED.2)</td>
</tr>
<tr>
<td>● Solving systems of equations (NC.M1.A-REI.6)</td>
</tr>
<tr>
<td>● Recognize the domain of sequences as integers (NC.M1.F-IF.3)</td>
</tr>
</tbody>
</table>

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<td>As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.</td>
</tr>
<tr>
<td>Students should be able to justify their use of a representation.</td>
</tr>
</tbody>
</table>

\[
\text{Solution: The area of the rectangle is } (x + 5)(x + 4) = x^2 + 9x + 20. \text{ Students should recognize } (x + 5) \text{ as the length of the modified rectangle and } (x + 4) \text{ as the width. Students can also interpret } x^2 + 9x + 20 \text{ as the sum of the three areas (a square with side length } x, \text{ a rectangle with side lengths } 9 \text{ and } x, \text{ and another rectangle with area 20 that have the same total area as the modified rectangle.)}
\]

\[
\text{Example: Given that income from a concert is the price of a ticket times each person in attendance, consider the equation } I = 4000p - 250p^2 \text{ that represents income from a concert where } p \text{ is the price per ticket. What expression could represent the number of people in attendance?}
\]

\[
\text{Solution: The equivalent factored form, } p (4000 - 250p), \text{ shows that the income can be interpreted as the price times the number of people in attendance based on the price charged. Students recognize } (4000 - 250p) \text{ as a single quantity for the number of people in attendance.}
\]
### Standards Addressed in this Unit

- Relate domain and range of a function to its graph (NC.M1.F-IF.5)
- Calculate the average rate of change (NC.M1.F-IF.6)
- Use equivalent forms of quadratic and exponential function to reveal key features (NC.M1.F-IF.8a, NC.M1.F-IF.8b)
- Compare key features of two functions in different representations (NC.M1.F-IF.9)
- Build functions that describe a relationship between two quantities (NC.M1.F-BF.1a, NC.M1.F-BF.1b)
- Identify situations that can be modeled with linear and exponential functions (NC.M1.F-LE.1)
- Interpret the parameters of a linear and exponential function in context (NC.M1.F-LE.5)

### New Vocabulary:

- Exponential function
- Quadratic function

### Mastering the Standard

#### Comprehending the Standard

<table>
<thead>
<tr>
<th>Activity</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students should identify the key features of the three function families covered in Math 1: linear, quadratic, and exponential.</td>
<td>Students should be able to identify the key feature of linear, quadratic and exponential functions from the symbolic representation. <strong>Example:</strong> Without using the graphing capabilities of a calculator, sketch the graph of ( f(x) = x^2 + 7x + 10 ) and identify the x-intercepts, y-intercept, and the maximum or minimum point.</td>
</tr>
</tbody>
</table>
| Students should be aware of the key functions typically associated with each function type. Quadratic functions – domain & range, y-intercept, x-intercepts (zeros), intervals of increasing and decreasing, intervals of positive and negative values, maximums and minimums, and end behavior. | Students should be able to identify the key feature of linear, quadratic and exponential functions from the graphical representation. **Example:** Which of the following is the function graphed below? \( f(x) = 4x^2 - 8x + 7 \) \( A \) \( f(x) = x^2 + 7x + 3 \) \( B \) \( f(x) = 7x^2 - 4x + 3 \) \( C \) \( f(x) = 3x^2 + x + 7 \) \( D \) |}

#### Assessing for Understanding

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>It is important for students to begin developing an understanding of end behavior and interpreting mathematical notation (such as ( x \to \infty )). As students study end behavior of these function families, connect their mathematical thinking from “as we keep going out” or “as ( x ) gets really big” to “as ( x ) goes to infinity”.</td>
<td><strong>Example:</strong> Which of the following could be the function of a real variable ( x ) whose graph is shown below? Explain.</td>
</tr>
</tbody>
</table>
At the Math 1 level, students should **not** be exposed to finding the line of symmetry of a quadratic function using the formula $x = \frac{-b}{2a}$ unless it is developed conceptually. This concept should be developed with a study of the quadratic formula, which will be done in Math 2. If the students need to find the line of symmetry (not a requirement of Math 1), they can find the midpoint of the zeros of the function.

*This task could be modified for a Math 1 classroom to not use vertex form.*

**NC.M1.F-IF.9**

*Analyze functions using different representations.*

Compare key features of two functions (linear, quadratic, or exponential) each with a different representation (symbolically, graphically, numerically in tables, or by verbal descriptions).

### Pre-requisite

- Compare properties of two functions each represented in different ways (8.F.2)
- Formally define a function (NC.M1.F-IF.1)
- Identify key feature of graphs and tables of functions (NC.M1.F-IF.4)
- Identify and interpret key features of functions from different representations (NC.M1.F-IF.7)
- Rewrite quadratic functions to identify key features (NC.M1.F-IF.8a)
- Interpret and explain growth and decay rates for an exponential function (NC.M1.F-IF.8b)

### Concepts and Skills

<table>
<thead>
<tr>
<th>Function 1</th>
<th>Function 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1(x) = (x + 12)^2 + 4$</td>
<td>$f_2(x) = -(x - 2)^2 - 1$</td>
</tr>
<tr>
<td>$f_3(x) = (x + 18)^2 - 40$</td>
<td>$f_4(x) = (x - 12)^2 - 9$</td>
</tr>
<tr>
<td>$f_5(x) = -4(x + 2)(x + 3)$</td>
<td>$f_6(x) = (x + 4)(x - 6)$</td>
</tr>
<tr>
<td>$f_7(x) = (x - 12)(-x + 18)$</td>
<td>$f_8(x) = (24 - x)(40 - x)$</td>
</tr>
</tbody>
</table>

**The Standards for Mathematical Practices**

**Implementing the Standards for Mathematical Practice**

**Connections**

*Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.*

- 4 – Model with mathematics
- 5 – Use appropriate tools strategically

**Disciplinary Literacy**

*As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.*

Students should be able to justify their use of a representation to make the comparison.

*New Vocabulary: exponential function, quadratic function*
### Mastering the Standard

<table>
<thead>
<tr>
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<th>Assessing for Understanding</th>
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</thead>
</table>
| Students should compare two functions in two different forms. The function types may be the same (linear & linear) or different (linear & exponential), but the representations should be different (e.g. numerical & graphical). It is important to note that the point of this standard is not to have students simply translate one function into the same form as the other function when given in different forms. Students should be able to use appropriate tools to compare the key features of functions. | Students should compare two functions in two different forms. **Example:** Suppose Brett and Andre each throws a baseball into the air. The height of Brett's baseball is given by \( h(t) = -16t^2 + 79t + 6 \), where \( h \) is in feet and \( t \) is in seconds. The height of Andre's baseball is given by the graph below: Brett claims that his baseball went higher than Andre's, and Andre says that his baseball went higher.  
   a) Who is right?  
   b) How long is each baseball airborne?  
   c) Construct a graph of the height of Brett's throw as a function of time on the same set of axes as the graph of Andre's throw (if not done already), and explain how this can confirm your claims to parts (a) and (b).  

**Example:** Kevin compared the \( y \)-intercept of the graph of the function \( f(x) = 3x^2 + 5 \) to the \( y \)-intercept of the graph of the linear function that includes the points in the table to the right. What is the difference when the \( y \)-intercept of \( f(x) \) is subtracted from the \( y \)-intercept of \( g(x) \)? |

### NC.M1.F-LE.3
**Construct and compare linear and exponential models and solve problems.**

Compare the end behavior of linear, exponential, and quadratic functions using graphs and tables to show that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically.

#### The Standards for Mathematical Practices
**Implementing the Standards for Mathematical Practice**

**Connections**

*Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.*

4 – Model with mathematics

**Disciplinary Literacy**

*As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.*

New Vocabulary: exponential function, quadratic function

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NC Math 1 Unit 4 Introduction to Quadratic Functions and Equations

September 2017 Update - Feedback

Return to: Standards Addressed in this Unit

121
### Mastering the Standard

**Comprehending the Standard**

Students experiment with the function types to build an understanding that the average rate of change over an interval for an exponential function will eventually surpass the rate of change of a linear or quadratic function over the same interval.

Students should be able to demonstrate this using various representations.

It is important for students to begin developing an understanding of end behavior and interpreting mathematical notation (such as $x \rightarrow \infty$). As students study end behavior of these function families, connect their mathematical thinking from “as we keep going out” or “as $x$ gets really big” to “as $x$ goes to infinity”.

**Assessing for Understanding**

Students should realize that an exponential function is eventually always bigger than a linear or quadratic function.

**Example:** Kevin and Joseph each decide to invest $100. Kevin decides to invest in an account that will earn $5 every month. Joseph decided to invest in an account that will earn 3% interest every month.

a) Whose account will have more money in it after two years?

b) After how many months will the accounts have the same amount of money in them?

c) Describe what happens as the money is left in the accounts for longer periods of time.

**Example:** Using technology, determine the average rate of change of the following functions for intervals of their domains in the table.

<table>
<thead>
<tr>
<th>Functions</th>
<th>Average rate of change</th>
<th>0 ≤ x ≤ 10</th>
<th>Average rate of change</th>
<th>10 ≤ x ≤ 20</th>
<th>Average rate of change</th>
<th>20 ≤ x ≤ 30</th>
<th>Average rate of change</th>
<th>30 ≤ x ≤ 40</th>
<th>Average rate of change</th>
<th>40 ≤ x ≤ 50</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x) = x^2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f(x) = 1.17^x$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a) When does the average rate of change of the exponential function exceed the average rate of change of the quadratic function?

b) Using a graphing technology, graph both functions. How do the average rates of change in your table relate to what you see on the graph?

Note: You can use the information in your table to determine how to change the setting to see where the functions intersect.

c) In your graphing technology, change the first function to $f(x) = 10x^2$ and adjust the settings to see where the functions intersect. What do you notice about the rates of change interpreted from the graph?

d) Make a hypothesis about the rates of change about polynomial and exponential function. Try other values for the coefficient of the quadratic function to support your hypothesis.
NC.M1.F-IF.6

*Interpret functions that arise in applications in terms of the context.*

Calculate and interpret the average rate of change over a specified interval for a function presented numerically, graphically, and/or symbolically.

### Concepts and Skills

#### Pre-requisite
- Determine and interpret the rate of change of a linear function (8.F.4)
- Describe qualitatively the functional relationship between two quantities and sketch a graph from a verbal description (8.F.5)

#### Connections
- Interpret key features of graphs and tables (NC.M1.F-IF.4)
- Analyze linear, quadratic and exponential functions by generating different representations (NC.M1.F-IF.7)

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### The Standards for Mathematical Practices

#### Implementing the Standards for Mathematical Practice

#### Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

#### Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary: average rate of change

### Mastering the Standard

#### Comprehending the Standard

Students calculate the average rate of change of a function given a graph, table, and/or equation.

The average rate of change of a function $y = f(x)$ over an interval $a \leq x \leq b$ is

$$\text{Average rate of change} = \frac{f(b) - f(a)}{b - a}.$$

This standard is more than just slope. It is asking students to find the average rate of change of any function over any given interval. Be sure to include multiple representations (numerically, graphically, or symbolically) of functions for students to work with.

It is an important connection for further learning.

---

#### Assessing for Understanding

**Example:** Find the average rate of change of each of the following functions over the interval $1 \leq x \leq 5$.

- $f(x) = 3x - 7$
- $g(x) = x^2 + 2x - 5$
- $h(x) = 3(2)^x$

**Example:** The table below shows the average weight of a type of plankton after several weeks.

What is the average rate of change in weight of the plankton from week 8 to week 12?

<table>
<thead>
<tr>
<th>Time (weeks)</th>
<th>Weight (ounces)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>0.04</td>
</tr>
<tr>
<td>9</td>
<td>0.07</td>
</tr>
<tr>
<td>10</td>
<td>0.14</td>
</tr>
<tr>
<td>11</td>
<td>0.25</td>
</tr>
<tr>
<td>12</td>
<td>0.49</td>
</tr>
</tbody>
</table>

A) 0.0265 ounce per week  
B) 0.0375 ounce per week  
C) 0.055 ounce per week  
D) 0.1125 ounce per week

*(NCDPI Math 1 released EOC #21)*

**Example:** The table below shows the temperature, $T$, in Tucson, Arizona $t$ hours after midnight.
courses that students recognize that linear functions have consistent average rate of change over any interval, while functions like quadratics and exponentials do not have constant rates of change due to their curvature.

When does the temperature decrease the fastest: between midnight and 3 a.m. or between 3 a.m. and 4 a.m.?

<table>
<thead>
<tr>
<th>t (hours after midnight)</th>
<th>0</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>T (temp. in °F)</td>
<td>85</td>
<td>76</td>
<td>70</td>
</tr>
</tbody>
</table>

(https://www.illustrativemathematics.org/content-standards/HSF/IF/B/tasks/1500)

NC.M1.F-IF.8a

*Analyze functions using different representations.*

Use equivalent expressions to reveal and explain different properties of a function.

a. Rewrite a quadratic function to reveal and explain different key features of the function

### Concepts and Skills

**Pre-requisite**

- Interpret parts of expressions in context (NC.M1.A-SSE.1a, NC.M1.A-SSE.1b)
- Factor to reveal key features (NC.M1.A-SSE.3)
- Operations with polynomials (NC.M1.A-APR.1)
- Understand the relationship between linear factors and zeros (NC.M1.A-APR.3)
- Formally define a function (NC.M1.F-IF.1)

**Connections**

- Identify key feature of graphs and tables of functions (NC.M1.F-IF.4)
- Identify and interpret key features of functions from different representations (NC.M1.F-IF.7)
- Compare key features of two functions in different representations (NC.M1.F-IF.9)

**Disciplinary Literacy**

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary: quadratic function

### The Standards for Mathematical Practices

**Implementing the Standards for Mathematical Practice**

**Connections**

- Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.
- 4 – Model with mathematics
- 5 – Use appropriate tools strategically

### Mastering the Standard

**Comprehending the Standard**

This set of standards requires that students rewrite expressions of quadratic and exponential functions to reveal key features of their graphs.

**Assessing for Understanding**

Students should be able to factor quadratic expressions to find key features of the quadratic function.
This is the “why” behind rewriting an expression where NC.M1.A-SSE.1 is the “how”. Therefore, these two standards should be taught together. This standard should also tie to the key features of graphs in NC.M1.F.IF.7

At the Math 1 level, students only know two forms of quadratics; standard and factored. Students SHOULD NOT complete the square or write a quadratic in vertex form. Therefore, other methods for finding the vertex should be used, such as calculating the midpoint between two zeros to find the x-value of the vertex and using function notation to determine the y-value of the vertex. Using a graphing utility to analyze key features of a quadratic function may be necessary.

At the Math 1 level, students should not be exposed to finding the line of symmetry of a quadratic function using the formula \( x = \frac{-b}{2a} \), unless it is developed conceptually. This concept can be developed with a study of the quadratic formula in Math 2. If the students need to find the line of symmetry (not a requirement of Math 1), they can find the midpoint of the zeros of the function.

The typical key features of a quadratic functions are: domain and range, y-intercept, x-intercepts (zeros), intervals of increasing and decreasing, intervals of positive and negative values, maximums and minimums, and end behavior.

**Example:** Suppose \( h(t) = -5t^2 + 10t + 15 \) is the height of a diver above the water (in meters), \( t \) seconds after the diver leaves the springboard.

- a) How high above the water is the springboard? Explain how you know.
- b) When does the diver hit the water?
- c) At what time on the diver’s descent toward the water is the diver again at the same height as the springboard?
- d) When does the diver reach the peak of the dive?

(https://www.illustrativemathematics.org/content-standards/HSF/IF/C/8/tasks/375)

**Example:** The function \( f(t) = -5t^2 + 20t + 60 \) models the approximate height of an object \( t \) seconds after it is launched. How many seconds does it take the object to hit the ground?

(NCDPI Math 1 released EOC #9)

**Example:** Suppose that the equation \( V = 20.8x^2 - 458.3x + 3500 \) represents the value of a car from 1964 to 2002. What year did the car have the least value? (\( x = 0 \) in 1964)

- A) 1965
- B) 1970
- C) 1975
- D) 1980

(NCDPI Math 1 released EOC #19)
NC.M1.A-REI.10
*Represent and solve equations and inequalities graphically*
Understand that the graph of a two variable equation represents the set of all solutions to the equation.

### Concepts and Skills

**Pre-requisite**
- Use substitution to determine if a number if a solution (6.EE.5)
- Graphing lines (8.EE.5, 8.EE.6, 8.F.3)
- Analyze and solve pairs of simultaneous linear equations by graphing and substitution (8.EE.8)
- Understanding functions as a rule that assigns each input with exactly one output (8.F.1)

**Connections**
- Creating and graphing two-variable equations (NC.M1.A-CED.2)
- Understanding that the relationship between the solution of system of equations and the associated equation (NC.M1.A-REI.11)
- Representing the solutions to linear inequalities (NC.M1.A-REI.12)
- Relating a function to its graph, domain and range of a function (NC.M1.F-IF.1, NC.M1.F-IF.2, NC.M1.F-IF.5)

### The Standards for Mathematical Practices

**Implementing the Standards for Mathematical Practice**

**Connections**
*Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.*

**Disciplinary Literacy**
*As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.*
Students should be able to discuss the solutions to a two variable equation and the link to a function.

### Mastering the Standard

<table>
<thead>
<tr>
<th>Comprehending the Standard</th>
<th>Assessing for Understanding</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students understand that the graph of an equation is the set of all ordered pairs that make that equation a true statement. This standard contains no limitation and so applies to all function types, including those functions that a student cannot yet algebraically manipulate.</td>
<td>Students should be able to assess if a point is a solution to an equation.</td>
</tr>
</tbody>
</table>

*September 2017 Update* - [Feedback](#)
y) on the graph of an equation represents all values for x and y that make the equation true.

**Example:** Without graphing, determine if the ordered pair \((2, -15)\) is on the graph of \(y = 3x^2 + 2x - 1\). Explain.

**Example:** Given the function to the right, determine if the following points are solutions and explain each answer.

- a) \((2, 1)\)
- b) \((3, 8)\)
- c) \((-1, -4)\)

Note: \((y < y^2 - y - 1)\) is the inequality of the graph

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**NC.M1.F-IF.2**

_Understand the concept of a function and use function notation._

Use function notation to evaluate linear, quadratic, and exponential functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

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### Concepts and Skills

<table>
<thead>
<tr>
<th>Pre-requisite</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Use substitution to determine if a number if a solution (6.EE.5)</td>
</tr>
<tr>
<td>- Interpret parts of expressions in context (NC.M1.A-SSE.1a, NC.M1.A-SSE.1b)</td>
</tr>
<tr>
<td>- Every point on the graph of an equation is a solution to the equation (NC.M1.A-REI.10)</td>
</tr>
<tr>
<td>- Define a function and use functions notation (NC.M1.F-IF.1)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Connections</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Creating and solving one variable equations (NC.M1.A-CED.1)</td>
</tr>
<tr>
<td>- Creating and graphing two variable equations (NC.M1.A-CED.2)</td>
</tr>
<tr>
<td>- Every point on the graph of an equation is a solution to the equation</td>
</tr>
</tbody>
</table>

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### The Standards for Mathematical Practices

**Implementing the Standards for Mathematical Practice**

**Connections**

_Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard._

1 – Make sense of problems and persevere in solving them

**DisciplinaryLiteracy**

_As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication._

Students should be able to discuss the domain, range, input, output and the...
NC Math 1 Unit 4 Introduction to Quadratic Functions and Equations

Standards Addressed in this Unit

- NC.M1.A-REI.10
  - Comparing the end behavior of functions (NC.M1.F-LE.3)

New Vocabulary: exponential function, quadratic function

Mastering the Standard

Comprehending the Standard
Students should be fluent in using function notation to evaluate a linear, quadratic, and exponential function. Students should be able to interpret statements in function notation in contextual situations.

Assessing for Understanding

Students should be able to evaluate functions written in function notation.

**Example:** Evaluate $f(2)$ for the function $f(x) = 3x^2 + 2x - 5$.

Students should be able to evaluate functions and interpret the result in a context.

**Example:** Suppose Matthew throws a baseball into the air. The height of the baseball at any given time, $t$, can be modeled by the function $h(t) = -16t^2 + 65t + 5$.

a) What is the height of the baseball after 2 seconds?

b) If $h(t) = 54$, what does this mean in context of the baseball scenario?

NC.M1.A-CED.2

*Create equations that describe numbers or relationships.*

Create and graph equations in two variables to represent linear, exponential, and quadratic relationships between quantities.

**Pre-requisite**

- Construct a linear function that models the relationship between two quantities (8.F.4)
- Graph linear equations (8.EE.6)
- The graph of a function is the set of ordered pairs consisting of input and a corresponding output (8.F.1)
- Understand that the graph of a two-variable equation represents the set of all solutions to the equation (NC.M1.A-REI.10)

**Connections**

- Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.
  - 4 – Model with mathematics
  - 6 – Attend to precision
  - 7 – Look for and make use of structure

September 2017 Update - Feedback
### Connections

- Interpret parts of an expression in context (NC.M1.A-SSE.1a,b)
- Creating linear equations for a system (NC.M1.A-CED.3)
- Solving for a variable of interest in a formula (NC.M1.A-CED.4)
- The graph a function \( f \) is the graph of the equation \( y = f(x) \) (NC.M1.F-IF.1)
- Interpret a function’s domain and range in context (NC.M1.F-IF.5)
- Identify key features of linear, exponential and quadratic functions (NC.M1.F-IF.7)
- Building a function through patterns or by combining other functions (NC.M1.F-BF.1a, NC.M1.F-BF.1b)

### Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to describe the origins of created equations and demonstrate its relation to the context.

New Vocabulary: exponential function, quadratic function

### Mastering the Standard

#### Comprehending the Standard

Students create equations in two variables. Students graph equations on coordinate axes with labels and scales clearly labeling the axes defining what the values on the axes represent and the unit of measure. Students also select intervals for the scale that are appropriate for the context and display adequate information about the relationship. Students interpret the context and choose appropriate minimum and maximum values for a graph.

In Math I, focus on linear, exponential and quadratic contextual situations for students to create equations in two variables.

#### Assessing for Understanding

Students should be able to create two variable equations from various representations, such as verbal descriptions, and use them to solve problems.

**Example:** The larger leg of a right triangle is 3 cm longer than its smaller leg. The hypotenuse is 6 cm longer than the smaller leg. How many centimeters long is the smaller leg?

**Example:** The floor of a rectangular cage has a length 4 feet greater than its width, \( w \). James will increase both dimensions of the floor by 2 feet. Which equation represents the new area, \( N \), of the floor of the cage?

- a) \( N = w^2 + 4w \)
- b) \( N = w^2 + 6w \)
- c) \( N = w^2 + 6w + 8 \)
- d) \( N = w^2 + 8w + 12 \)

Students should be able to create two variable equations, graph the relationship, and use graph to recognize key feature of the graph.

**Example:** Misha has a new rabbit that she named “Wascal.” She wants to build Wascal a pen, so that the rabbit has space to move around safely. Misha has purchased a 72 foot roll of fencing to build a rectangular pen.

- a) If Misha uses the whole roll of fencing, what are some of the possible dimensions of the pen?
  - Write a model for the area of the rectangular pen in terms of the length of one side. Include both an equation and a graph.
  - What are the dimensions of the pen that would allow Wascal the most area to run around? How do you know?

[www.mathematicsvisionproject.org](http://www.mathematicsvisionproject.org)
NC.M1.A-CED.1

*Create equations that describe numbers or relationships.*
Create equations and inequalities in one variable that represent linear, exponential, and quadratic relationships and use them to solve problems.

### Concepts and Skills

#### Pre-requisite
- Create two-step linear equations and inequalities from a context (7.EE.4)

#### Connections
- Interpret parts of an expression in context (NC.M1.A-SSE.1a,b)
- Justify a chosen solution method and each step of a that process (NC.M1.A-REI.1)
- Solve linear, exponential and quadratic equations using tables and graphs (NC.M1.A-REI.11)
- Represent the solutions of linear inequalities on a graph (NC.M1.A-REI.12)

### The Standards for Mathematical Practices

#### Implementing the Standards for Mathematical Practice

#### Connections

*Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.*

- 4 – Model with mathematics
- 7 – Look for and make use of structure

#### Disciplinary Literacy

*As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.*

Students should be able to describe the origins of created equations and inequalities and demonstrate its relation to the context.

New Vocabulary: exponential function, quadratic function
Comprehending the Standard

Students create equations and inequalities in one-variable and use them to solve problems.

In Math I, focus on linear, quadratic, and exponential contextual situations that students can use to create equations and inequalities in one variable and use them to solve problems. It is also important to note that equations can be created from an associated function when a given value is substituted in for either the independent or dependent variable.

After the students have created an equation, they can use other representations to assist in solving problems, such as graphs and tables.

For quadratic inequalities, the focus of this standard is to create the inequality and use that inequality to solve a problem. Solving these inequalities algebraically is not part of the standard. Once a student has the inequality, the student can use a table or graph to find a solution to the problem.

Assessing for Understanding

Students should be able to create an equation from a function and use the equation to solve problems.

Example: A ball thrown vertically upward at an initial velocity of \( v_0 \) ft/sec rises a distance \( d \) feet in \( t \) seconds, given by \( d = 6 + v_0 t - 16t^2 \).

Write an equation whose solution is:

a) The time it takes a ball thrown at a speed of 88 ft/sec to rise 20 feet.

b) The speed with which the ball must be thrown to rise 20 feet in 2 seconds. (https://www.illustrativemathematics.org/content-standards/HSA/CED/A/2/tasks/437)

Example: A ball thrown vertically upward at an initial velocity of 88 ft/sec rises a distance \( d \) feet in \( t \) seconds, given by \( d = 6 + 88t - 16t^2 \).

a) Write an inequality whose solution represents when the ball would be at least 78 ft above the ground.

b) Use the table to the right to find when the ball would be at least 78 ft above the ground.

Students should be able to create equations from various representations, such as verbal descriptions, and use them to solve problems. Students should be able to create inequalities and use those inequalities to solve problems. (Students are not expected to solve quadratic and exponential inequalities algebraically. Students should use technology, tables and graphs to solve problems.)

Example: Stephen wants to create a landscaping feature in the shape of a parallelogram in his yard. Stephen has 200 square feet of mulch available for the project. To be most pleasing to the eye, he decides that he wants the length of the parallelogram to be 3 more than twice the width, measured in feet. If Stephen intends to cover the entire landscape feature in mulch, what can the width of the parallelogram be?
NC.M1.A-SSE.3
Write expressions in equivalent forms to solve problems.
Write an equivalent form of a quadratic expression by factoring, where \( a \) is an integer of the quadratic expression, \( ax^2 + bx + c \), to reveal the solutions of the equation or the zeros of the function the expression defines.

### Concepts and Skills

**Pre-requisite**
- Factoring and expanding linear expressions with rational coefficients (7.EE.1)
- Understand that rewriting expressions into equivalent forms can reveal other relationships between quantities (7.EE.2)

**Connections**
- Interpreting the factors in context (NC.M1.A-SSE.1b)
- Understanding the relationship between factors, solutions, and zeros (NC.M1.A-APR.3)
- Solving quadratic equations (NC.M1.A-REI.4)
- Rewriting quadratic functions into different forms to show key features of the function (NC.M1.F-IF.8a)

### The Standards for Mathematical Practices

**Implementing the Standards for Mathematical Practice**

**Connections**
Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

- 4 – Model with mathematics
- 7 – Look for and make use of structure.

**Disciplinary Literacy**
As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to compare and contrast the zeros of a function and the solutions of a function.

New Vocabulary: quadratic expression, zeros, linear factors

### Mastering the Standard

**Comprehending the Standard**
Students factor a quadratic in the form \( ax^2 + bx + c \) where \( a \) is an integer in order to reveal the zeros of the quadratic function.

Students use the linear factors of a quadratic function to explain the meaning of the zeros of quadratic functions and the solutions to quadratic equations in a real-world problem.

**Assessing for Understanding**
Students should understand that the reasoning behind rewriting quadratic expressions into factored form is to reveal different key features of a quadratic function, namely the zeros/x-intercepts.

**Example:** The expression \(-4x^2 + 8x + 12\) represents the height of a coconut thrown from a person in a tree to a basket on the ground where \( x \) is the number of seconds.

a) Rewrite the expression to reveal the linear factors.
b) Identify the zeros and intercepts of the expression and interpret what they mean in regard to the context.
c) How long is the ball in the air?

**Example:** Part A: Three equivalent equations for \( f(x) \) are shown. Select the form that reveals the zeros of \( f(x) \) without changing the form of the equation.

\[
f(x) = -2x^2 + 24x - 54 \quad f(x) = -2(x - 3)(x - 9) \quad f(x) = -2(x - 6)^2 + 18
\]
Part B: Select all values of $x$ for which $f(x) = 0$.

$-54, -18, -9, -6, -3, 0, 3, 6, 9, 18, 54$

(from the Smarter Balanced Assessment Consortium)

Students should understand that the reasoning behind rewriting quadratic expressions into factored form is to reveal the solutions to quadratic equations.

**Example:** A vacant rectangular lot is being turned into a community vegetable garden with a uniform path around it. The area of the lot is represented by $4x^2 + 40x - 44$ where $x$ is the width of the path in meters. Find the width of the path surrounding the garden.

---

**NC.M1.A-REI.4**

*Solve equations and inequalities in one variable.*

Solve for the real solutions of quadratic equations in one variable by taking square roots and factoring.

**Concepts and Skills**

**Pre-requisite**

- Factor linear expressions with rational coefficients (7.EE.1)
- Use square root to represent solutions to equations of the form $x^2 = p$, where $p$ is a positive rational number; evaluate square roots of perfect squares (8.EE.2)
- Factor a quadratic expression to reveal the solution of a quadratic equation (NC.M1.A-SSE.3)
- Understand the relationship between linear factors and solutions (NC.M1.A-APR.3)

**Connections**

- Create one variable quadratic equations and inequalities and solve (NC.M1.A-CED.1)
- Justify a solution method and each step in the solution process (NC.M1.A-REI.1)

**Implementing the Standards for Mathematical Practice**

**Connections**

*Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.*

6 – Attend to precision
7 – Look for a make use of structure

**Disciplinary Literacy**

*As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.*

Students should be able to discuss their solution method and the steps in the solving process and should be able to interpret the solutions in context.

New Vocabulary: quadratic equation

---

**Mastering the Standard**

**Comprehending the Standard**

Students should focus on quadratics with one or two real solutions that can be solved by factoring or taking the square root.

**Assessing for Understanding**

Students should be able to solve quadratic equations using square root as the inverse operation.
This standard gives the algebraic reasoning of how to solve a quadratic equation to find the \(x\) -intercepts and zeroes that exist on the graph of the associated quadratic function. This standard gives the “how” to solve a quadratic equation while NC.M1.A-APR.3 gives the “why”. Therefore, these two standards should be taught together. Students should be able to use the structure of the quadratic equation to determine whether to solve by using the square root as an inverse operation or by factoring. When solving using the square root, students are only expected to evaluate perfect squares. All other square root solutions should either be left in square root form or estimated appropriately based on the context. Therefore, solving using the quadratic formula is not expected at this level.

<table>
<thead>
<tr>
<th>Example: Solve:</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) ( x^2 = 49 )</td>
</tr>
<tr>
<td>b) ( 3x^2 + 9 = 72 )</td>
</tr>
</tbody>
</table>

Students should be able to solve quadratic equations using factoring. **Example:** Solve: \( 6x^2 + 13x = 5 \)

Students should be able to discuss their chosen solution method. **Example:** Stephen and Brianna are solving the quadratic equation, \((x - 4)^2 - 25 = 0\), in a classroom activity. Stephen believes that the equation can be solving using a square root. Brianna disagrees, saying that it can be solve using by factoring. Who is correct? Be prepared to defend your position.

NC.M1.A-APR.3

**Understand the relationship between zeros and factors of polynomials.**

Understand the relationships among the factors of a quadratic expression, the solutions of a quadratic equation, and the zeros of a quadratic function.

### Concepts and Skills

**Pre-requisite**
- Understand that is the product is zero, at least one of the factors is zero (3.OA.7)

**Connections**
- Factor quadratic expressions to reveal zeros of functions and solutions to equations (NC.M1.A-SSE.3)
- Justify the steps in solving a quadratic equation (NC.M1.A-REI.1)
- Solving quadratic equations (NC.M1.A-REI.4)
- Factor quadratic functions to reveal key features (NC.M1.F-IF.8)

### The Standards for Mathematical Practices

**Implementing the Standards for Mathematical Practice**

**Connections**

*Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.*

2 – Reason abstractly and quantitatively

7 – Look for and make use of structure

**Disciplinary Literacy**

*As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.*

Students should be able to compare solutions functions to solutions of equations. New Vocabulary: quadratic expression, quadratic equation, quadratic function, zeroes, linear factors, roots

### Mastering the Standard

<table>
<thead>
<tr>
<th>Comprehending the Standard</th>
<th>Assessing for Understanding</th>
</tr>
</thead>
</table>

September 2017 Update - Feedback
The focus of this standard is for students to use the multiplicative property of zero to create linear factors given a quadratic equation, and to solve those linear factors to find a zero of a function or a solution of an equation. This standard should be taught with NC.M1.A-SSE.3 and NC.M1.A-REI.1.

Students can find the solutions of a factorable quadratic equation and use the roots to sketch its \( x \)-intercepts on the graph.

Students should be able to explain how they go from factored form to identifying the zeros of the function.

**Example:** Given the function \( y = 2x^2 + 6x - 3 \), list the zeroes of the function and sketch its graph.

**Example:** Sketch the graph of the function \( f(x) = (x + 5)^2 \). How many zeros does this function have? Explain. Note: It is a common error for students to assume that the solution or zero of linear factor, \((x - b)\), will always be the opposite of the constant term, \(b\). If this is noticed, be sure to include examples in which \(a \neq 1\).

**Example:** Which of the following are the solutions to the equation \( x^2 - 13x = 30 \)?
- A) \( x = -10 \& 3 \)
- B) \( x = 10 \& -3 \)
- C) \( x = -15 \& 2 \)
- D) \( x = 15 \& -2 \)

**Example:** Which of the following has the largest \(x\)-intercept?
- A) \( x^2 + 4x - 12 \)
- B) \((x + 2)(x - 5)\)
- C) \((x - 1)^2 - 4\)

Students should understand the relationship between zeros/solutions and the quadratic expression.

**Example:** If the zeros of a function are \( x = 2 \) and \( x = 7 \), what was the function? Could there be more than one answer?

**Example:** Based on the graph to the right, which of the following functions could have produced the graph?
- A) \( f(x) = (x + 2)(x + 6) \)
- B) \( f(x) = (x - 2)(x + 6) \)
- C) \( f(x) = (2 - x)(6 - x) \)
- D) \( f(x) = (2 + x)(6 - x) \)

**NC.M1.A-REI.1**

*Understand solving equations as a process of reasoning and explain the reasoning.*

Justify a chosen solution method and each step of the solving process for linear and quadratic equations using mathematical reasoning.

**Concepts and Skills**

**Pre-requisite**
- Students have been using properties of operations and equality throughout middle school. (6.EE.3, 7.EE.1, 7.EE.4). This is the first time that justification is required by a content standard.
- Solve multi-step equations (8.EE.7)

The Standards for Mathematical Practices

**Implementing the Standards for Mathematical Practice**

**Connections**
*Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.*

3 – Construct viable arguments and critique the reasoning of others
### Connections

- Understand the relationship between factors of a quadratic equation and the solution of the equation (NC.M1.A-APR.3)
- Create and solve one variable linear and quadratic equations (NC.M1.A-CED.1)
- Solve for a quantity of interest in a formula (NC.M1.A-CED.4)

### Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to defend their method of solving an equation and each step of the solving process.

New Vocabulary: quadratic equation

### Mastering the Standard

<table>
<thead>
<tr>
<th>Comprehending the Standard</th>
<th>Assessing for Understanding</th>
</tr>
</thead>
</table>
| When solving equations, students will use the properties of equality to justify and explain each step obtained from the previous step, assuming the original equation has a solution, and develop an argument that justifies their method. Properties of operations can be used to change expressions on either side of the equation to equivalent expressions. In the properties of equality, adding the same term to both sides of an equation or multiplying both sides by a non-zero constant produces an equation with the same solutions. Students do not have to name the property, but can describe the property using mathematical reasoning. *For example:* Transforming $2x - 5 = 7$ to $2x = 12$ is possible because $5 = 5$, so adding the same quantity to both sides of an equation makes the resulting equation true as well. *Knowing this is the Addition Property of Equality is not the point of this standard.* | Students should be able to justify a chosen solution method and justify each step in the process. This would be a good opportunity to discuss efficiency. **Example:** Below are two methods for solving the equation $5x^2 + 10 = 90$. Select one of the solution methods and construct a viable argument for the use of the method.  

\[
\begin{align*}
5x^2 + 10 &= 90 - 10 \Rightarrow 10 \\
5x^2 &= 80 \\
x &= \pm \sqrt{16} \\
x &= 4 \text{ or } x = -4
\end{align*}
\]

\[
\begin{align*}
5x^2 + 10 &= 90 - 90 \Rightarrow 90 \\
5x^2 &= 80 \\
5(x^2 - 16) &= 0 \\
x+4 = 0 \text{ or } x - 4 = 0 \\
x &= 4 \text{ or } x = -4
\end{align*}
\]

---

**NC.M1.A-REI.11**  
*Represent and solve equations and inequalities graphically*  
Build an understanding of why the $x$-coordinates of the points where the graphs of two linear, exponential, or quadratic equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$ and approximate solutions using a graphing technology or successive approximations with a table of values.
Concepts and Skills

Pre-requisite
- Solving multi-step linear equations (8.EE.7)
- Analyze and solve pairs of simultaneous linear equations by graphing and substitution (8.EE.8)
- Understand every point on a graph is a solution to its associated equation (NC.M1.A.REI.10)

Connections
- Creating and solving one variable equations and systems of equations (NC.M1.A.CED.1, NC.M1.A.CED.3)
- Solving systems of equations (NC.M1.A.REI.6)

The Standards for Mathematical Practices

Implementing the Standards for Mathematical Practice

Connections
- Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.
  4 – Model with mathematics
  6 – Attend to precision

Disciplinary Literacy
- As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New Vocabulary: exponential function, quadratic function

Mastering the Standard

Comprehending the Standard
For a complete understanding, students will need exposure to both parts of this standard.

First, students should be able to see the connection between graphs and tables of two functions, the points they have in common and the truthfulness of the equation.

For example:

<table>
<thead>
<tr>
<th>x</th>
<th>f(x) = 2x - 4</th>
<th>x</th>
<th>g(x) = \frac{2x}{2} + \frac{1}{2}</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-4</td>
<td>0</td>
<td>0.5</td>
</tr>
<tr>
<td>1</td>
<td>-2</td>
<td>2</td>
<td>0.5</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>4</td>
<td>2.5</td>
</tr>
</tbody>
</table>

Because \( f(x) = g(x) \) when \( x = 3 \), 3 is the solution to the equation \( 2x - 4 = \frac{x}{2} + \frac{1}{2} \).

(As an extension, students could write an inequality to describe the relationship)

Assessing for Understanding

Example: The functions \( f(m) = 18 + 0.4m \) and \( g(m) = 11.2 + 0.54m \) give the lengths of two different springs in centimeters, as mass is added in grams, \( m \), to each separately.

  a) Graph each equation on the same set of axes.
  b) What mass makes the springs the same length?
  c) What is the length at that mass?
  d) Write a sentence comparing the two springs.

Example: Solve the following equations by graphing. Give your answer to the nearest tenth.

  a) \( 3(2x) = 6x - 7 \)
  b) \( 10x + 5 = -x + 8 \)

Example: The population of a country is initially 2 million people and is increasing at 4% per year. The country’s annual food supply is initially adequate for 4 million people and is increasing at a constant rate adequate for an additional 0.5 million people per year.
In Math 1, students are expected to solve linear systems of equations algebraically. All other systems should be solved with technology, tables, and graphs.

Second, students should be able to use a system of equations to solve systems of equations.

For example:
Solve: \(3x^2 - 2x + 1 = \frac{1}{2}x + 5\)
Rewrite the equations as a system of equations:
\[
\begin{align*}
f(x) &= 3x^2 - 2x + 1 \\
g(x) &= \frac{1}{2}x + 5
\end{align*}
\]
Using technology, graph the equations and look for points of intersection, where the same \(x\) produces \(f(x) = g(x)\).

In Math 1, students are expected to solve linear equations using inverse operations and quadratic equations with square roots and factoring. In all other equations, such as exponential equations, solutions should be approximated with technology, tables and graphs.

NC.M1.F-IF.4
*Interpret functions that arise in applications in terms of the context.*
Interpret key features of graphs, tables, and verbal descriptions in context to describe functions that arise in applications relating two quantities, including: intercepts; intervals where the function is increasing, decreasing, positive, or negative; and maximums and minimums.

### Concepts and Skills

**Pre-requisite**

- Describe quantitatively the functional relationship between two quantities by analyzing a graph (8.F.5)
- Define a function and use functions notation (NC.M1.F-IF.1)
- Evaluating functions (NC.M1.F-IF.2)

**Connections**

- Interpret parts of expressions in context (NC.M1.A-SSE.1a, NC.M1.A-SSE.1b)

### The Standards for Mathematical Practices

**Implementing the Standards for Mathematical Practice**

**Connections**

*Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.*

4 – Model with mathematics

**Disciplinary Literacy**

*As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.*
## Standards Addressed in this Unit

- Relate domain and range of a function to its graph (NC.M1.F-IF.5)
- Calculate the average rate of change (NC.M1.F-IF.6)
- Use equivalent forms of quadratic and exponential function to reveal key features (NC.M1.F-IF.8a, NC.M1.F-IF.8b)
- Compare key features of two functions in different representations (NC.M1.F-IF.9)
- Identify situations that can be modeled with linear and exponential functions (NC.M1.F-LE.1)

### Students should be able to justify their identification of key features and interpret those key features in context.

**New Vocabulary:** maximum, minimum

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### Mastering the Standard

<table>
<thead>
<tr>
<th>Comprehending the Standard</th>
<th>Assessing for Understanding</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students should understand the key features of any contextual situation. For example, plots over time represent functions as do some scatterplots. These are often functions that “tell a story” hence the portion of the standard that has students sketching graphs given a verbal description. Students should have experience with a wide variety of these types of functions and be flexible in thinking about functions and key features using tables, graphs, and verbal descriptions. Students should understand the concept behind the key features (intercepts, increasing/decreasing, positive/negative, and maximum/minimum) for any given graph, not just “function families”. This means that students should be asked to work with graphical and tabular representations of functions that the student could not solve or manipulate algebraically. By contrast, NC.M1.F-IF.7, has students work with specific functions in which students have the ability to use algebraic manipulation to identify additional key features.</td>
<td>Students should be able to identify and interpret key features of quadratic functions from graphs, tables or verbal descriptions. <strong>Example:</strong> Suppose Brett and Andre each throw a baseball into the air. The height of Brett’s baseball is given by ( h(t) = -16t^2 + 79t + 6 ), where ( h ) is in feet and ( t ) is in seconds. The height of Andre's baseball is given by the graph below: Interpret the ( x )-intercept, ( y )-intercept, and maximum in context of the baseball scenario.</td>
</tr>
</tbody>
</table>

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September 2017 Update - [Feedback]

NC Math 1 Unit 4 Introduction to Quadratic Functions and Equations

Return to: [Standards Addressed in this Unit](#)
NC.M1.F-BF.1b

Build a function that models a relationship between two quantities.
Write a function that describes a relationship between two quantities.

b. Build a function that models a relationship between two quantities by combining linear, exponential, or quadratic functions with addition and subtraction or two linear functions with multiplication.

The Standards for Mathematical Practices
Implementing the Standards for Mathematical Practice

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to justify their process of building a new function.
New Vocabulary: exponential function, quadratic function

Mastering the Standard

Comprehending the Standard

This standard is about building functions. In this part of the standard students should combine functions to represent a contextual situation.

This standard pairs well with Interpreting Functions standards, in that the purpose behind building a function is to then use that function to solve a problem.

The algebraic skills behind this standard occur in NC.M1.A-APR.1. This standard should be taught throughout the year as each new function family is added to the course.

Assessing for Understanding

Students should combine functions to represent a contextual situation.

Example: The floor of a rectangular cage has a length 4 feet greater than its width, \( w \). James will increase both dimensions of the floor by 2 feet. Which equation represents the new area, \( N \), of the floor of the cage?

A) \( N = w^2 + 4w \)
B) \( N = w^2 + 6w \)
C) \( N = w^2 + 6w + 8 \)
D) \( N = w^2 + 8w + 12 \)

(NCDPI Math 1 released EOC #5)
NC.M1.F-IF.5  
*Interpret functions that arise in applications in terms of the context.* 
Interpret a function in terms of the context by relating its domain and range to its graph and, where applicable, to the quantitative relationship it describes.

### Concepts and Skills

#### Pre-requisite
- In middle school, students only informally considered restrictions to the domain and range based on context, such as understanding that measurements cannot be negative.
- Interpret parts of expressions in context (NC.M1.A-SSE.1a, NC.M1.A-SSE.1b)
- Every point on the graph of an equation is a solution to the equation (NC.M1.A-REI.10)
- Formally define a function (NC.M1.F-IF.1)
- Evaluating functions and interpret in context (NC.M1.F-IF.2)

#### Connections
- Recognize the domain of sequences (NC.M1.F-IF.3)
- Identify key feature of graphs and tables of functions (NC.M1.F-IF.4)
- Analyze linear, quadratic, and exponential functions to identify key features (NC.M1.F-IF.7)

### The Standards for Mathematical Practices

#### Implementing the Standards for Mathematical Practice

**Connections**

*Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.*

4 – Model with mathematics

**Disciplinary Literacy**

*As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.*

### Mastering the Standard

#### Comprehending the Standard

Students should be able to associate a reasonable domain and range to a graph as well as to a contextual situation. The domain of a graph should be taught in the context of the situation it represents. Graphs represented should be both discrete and continuous forms. Students do not need to know the terminology discrete and...

#### Assessing for Understanding

Students should be able to identify a reasonable domain and range to its graph as well as to a contextual situation. **Example:** Maggie tosses a coin off of a bridge into a stream below. The distance the coin is above the water is modeled by the equation $y = -16x^2 + 96x + 112$, where $x$ represents time in seconds. What is a reasonable domain for the function?
continuous, but they should be able to identify which is appropriate for each contextual situation.

**NC.M1.S-ID.8**

*Interpret linear models.*

Analyze patterns and describe relationships between two variables in context. Using technology, determine the correlation coefficient of bivariate data and interpret it as a measure of the strength and direction of a linear relationship. Use a scatter plot, correlation coefficient, and a residual plot to determine the appropriateness of using a linear function to model a relationship between two variables.

### Concepts and Skills

**Pre-requisite**
- Construct and interpret scatterplots for two-variable data and describe patterns of association (8.SP.1)
- Fit a regression line to linear data using technology (NC.M1.S-ID.6a)
- Assess linearity by analyzing residuals (NC.M1.S-ID.6b)

**Connections**
- Identify situations that can be modeled with linear and exponential functions, and justify the most appropriate model (NC.M1.F-LE.1)

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### The Standards for Mathematical Practices

**Implementing the Standards for Mathematical Practice**

**Connections**

*Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.*

**Vocabulary**

*As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication. The following vocabulary is new to this course and supported by this standard:*

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### Mastering the Standard

<table>
<thead>
<tr>
<th>Comprehending the Standard</th>
<th>Assessing for Understanding</th>
</tr>
</thead>
</table>
| In working with bivariate data in MS, students have previously. The correlation coefficient, $r$, is a measure of the strength and direction of a linear relationship between two quantities in a set of data. The magnitude (absolute value) of $r$ | Students can interpret the correlation coefficient. **Example:** The correlation coefficient of a given data set is 0.97. List three specific things this tells you about the data. Students recognize the strength of the association of two quantities based on the scatter plot. **Example:** Which correlation coefficient best matches each graph? Explain. | Return to: Standards Addressed in this Unit
indicates how closely the data points fit a linear pattern.

If $r = \pm 1$, all points fall exactly on a line. The sign of $r$ indicates the direction of the relationship. The closer $|r|$ is to 1, the stronger the correlation and the closer $|r|$ is to zero, the weaker the correlation.

**Instructions for TI-83 and TI-84 series calculators:**
1: Go to the [catalog]. Click → 2nd then 0.
2: Scroll down to → DiagnosticOn and press enter twice.

When ‘Done’ appears on the screen the diagnostics are on and the calculator should now calculate the correlation coefficient ($r$) automatically when linear regression is performed.

<table>
<thead>
<tr>
<th>Average weekly temperature</th>
<th># of Sno-cones sold</th>
</tr>
</thead>
<tbody>
<tr>
<td>68</td>
<td>500</td>
</tr>
<tr>
<td>74</td>
<td>600</td>
</tr>
<tr>
<td>74</td>
<td>700</td>
</tr>
<tr>
<td>80</td>
<td>800</td>
</tr>
<tr>
<td>82</td>
<td>1200</td>
</tr>
</tbody>
</table>

A. $r = \_\_\_\_$  
B. $r = \_\_\_\_$  
C. $r = \_\_\_\_$  

$r = -.48$  $r = .98$  $r = .88$  $r = -.17$  $r = 1$  $r = .31$  $r = -1$

Students will be able to analyze patterns in context between two variables and use graphing technology to determine whether a linear model is appropriate for the data.

**Example:** The following data set indicates the average weekly temperature and the number of sno-cones sold by Sno-Show Sno-cones each week in May for the temperatures noted.

- $a)$ Using technology, sketch a scatter plot of the data.
- $b)$ Determine a linear regression model that could represent the data shown.
- $c)$ Determine the correlation coefficient.
- $d)$ Determine the strength and direction of the linear relationship.
- $e)$ Create a residual plot. Is a linear model appropriate for the data shown? Explain.

**NOTE:** Remind students to turn the Diagnostics on in the graphing calculator so that the correlation coefficient ($r$) appears when the regression equation is calculated.
NC Math 1
Unit 5: Systems of Equations and Inequalities

12 Days Block Schedule
24 Days Traditional Schedule

RESEARCH BRIEF: Systems of Equations and Inequalities

Essential Questions:
● How is the solution of a system of equations represented on a graph, table, and in context of the scenario?
● How is the solution to a system of linear inequalities represented and what does it mean in context of the scenario?

Learning Outcomes

● Given a scenario, students will create a system of linear equations to model the scenario.
● Students will solve systems of linear equations using the table and graph on the graphing calculator.
● Students will solve systems of linear equations algebraically with substitution and elimination.
● Students will explain solutions to systems of linear equations in context.
● Students will approximate solutions of systems of linear, quadratic, and exponential functions using technology.
● Given a scenario, students will create a system of linear inequalities to model the scenario.

Student Objectives

● I will explore systems of linear equations in different representations.
● I will explain solutions to systems of linear equations in context.
● I will create a system of equations to model a scenario.
● I will be challenged to solve systems of linear equations algebraically.
● I will solve systems of equations (linear, exponential, and/or quadratic) using technology.
● I will explore systems of linear inequalities.
● I will solve systems of linear inequalities graphically with and without a graphing calculator.
- Students will solve systems of linear inequalities graphically without the use of a graphing calculator.
- Students will solve systems of linear inequalities using the table and graph on the graphing calculator.
- Students will explain solutions to systems of linear inequalities in context.

I will understand possible solutions to a system of linear inequalities in context.

Standards Addressed in this Unit

Create, solve, and interpret systems of equations in context.

<table>
<thead>
<tr>
<th>OCS Priority Standard(s)</th>
<th>Supporting Standard(s)</th>
</tr>
</thead>
</table>
| **NC.M1.A-REI.5**: Explain why replacing one equation in a system of linear equations by the sum of that equation and a multiple of the other produces a system with the same solutions. | **NC.M1.A-CED.3**: Create systems of linear equations and inequalities to model situations in context.  
**NC.M1.A-REI.6**: Solve systems of equations using tables, graphs, or algebraic methods (substitution and elimination) to find the approximate or exact solutions to systems of linear equations and interpret solutions in terms of a context.  
**NC.M1.A-REI.11**: Build and understanding of why the x-coordinates of the points where the graphs of two linear, exponential, or quadratic equations $y=f(x)$ and $y=g(x)$ intersect are the solutions of the equation $f(x) = g(x)$ and approximate solutions using a graphing technology or successive approximations with a table of values. |
Create, solve, and interpret systems of inequalities in context.

<table>
<thead>
<tr>
<th>OCS Priority Standard(s)</th>
<th>Supporting Standard(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>NC.M1.A-REI.12</strong>: Solve and represent the solutions of a linear inequality or a system of linear inequalities graphically as a region of the plane.</td>
</tr>
</tbody>
</table>
The Math Resource for Instruction - Customized for the Content of this Unit

NC.M1.A-CED.3
Create equations that describe numbers or relationships.
Create systems of linear equations and inequalities to model situations in context.

Concepts and Skills

Pre-requisite
- Understanding a system of equations (8.EE.8)
- Creating linear equations in two variables (NC.M1.A-CED.2)

Connections
- Interpret parts of an expression in context (NC.M1.A-SSE.1a,b)
- Use tables, graphs and algebraic methods to solve systems of linear equations (NC.M1.A-REI.6)
- Represent the solution to a system of linear inequalities as a region of the plane (NC.M1.A-REI.12)

The Standards for Mathematical Practices
Implementing the Standards for Mathematical Practice

Connections
Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.
4 – Model with mathematics

Disciplinary Literacy
As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.
Students should be able to describe the origins of created equations and demonstrate its relation to the context.

Mastering the Standard

Comprehending the Standard
Students create a system of linear equations and inequalities that model real world situations.
The expectation for this standard is to create a system of linear equations or a system on linear inequalities that model a contextual situation. The system should include inequalities that limit the domain and range if necessary.

Assessing for Understanding
Students should be able to write inequalities that describe the limitations from a context for a system of inequalities.
Example: A club is selling hats and jackets as a fundraiser. Their budget is $1500 and they want to order at least 250 items. They must buy at least as many hats as they buy jackets. Each hat costs $5 and each jacket costs $8.

a) Write a system of inequalities to represent the situation.
b) Graph the inequalities.
c) If the club buys 150 hats and 100 jackets, will the conditions be satisfied?
d) What is the maximum number of jackets they can buy and still meet the conditions?

Students should be able to write the system of equations based on context.
Linear programming is not the intent of this standard. While it may be an extension of this standard and could be used as application, it is not the expectation that students be fluent in maximizing or minimizing based on constraints.

**Example:** The only coins that Alexis has are dimes and quarters.
- Her coins have a total value of $5.80.
- She has a total of 40 coins.

Which of the following systems of equations can be used to find the number of dimes, \( d \), and the number of quarters, \( q \), Alexis has? [https://www.illustrativemathematics.org/content-standards/HSA/CED/A/3/tasks/220](https://www.illustrativemathematics.org/content-standards/HSA/CED/A/3/tasks/220)

**NC.M1.A-REI.5**

**Solve systems of equations.**

Explain why replacing one equation in a system of linear equations by the sum of that equation and a multiple of the other produces a system with the same solutions.

<table>
<thead>
<tr>
<th>Concepts and Skills</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pre-requisite</strong></td>
</tr>
<tr>
<td>- Analyze and solve pairs of simultaneous linear equations by graphing and substitution (8.EE.8)</td>
</tr>
<tr>
<td>- Operations with polynomials (NC.M1.A-APR.1)</td>
</tr>
<tr>
<td>- Justify steps in a solving process (NC.M1.A-REI.1)</td>
</tr>
<tr>
<td><strong>Connections</strong></td>
</tr>
<tr>
<td>- Solving systems of equations and inequalities (NC.M1.A-REI.6)</td>
</tr>
<tr>
<td>- Understand that all points on the graph of an equation is a solution to that equation (NC.M1.A-REI.10)</td>
</tr>
</tbody>
</table>

**The Standards for Mathematical Practices**

**Implementing the Standards for Mathematical Practice**

**Connections**

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

**Disciplinary Literacy**

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication. Students should be able to explain why the process of elimination works. New Vocabulary: elimination

**Mastering the Standard**

<table>
<thead>
<tr>
<th>Comprehending the Standard</th>
<th>Assessing for Understanding</th>
</tr>
</thead>
<tbody>
<tr>
<td>The focus of this standard is to explain a mathematical justification for the addition (elimination) method of solving systems of equations ultimately transforming a given system of two equations into a simpler equivalent system that has the same solutions as the original system. Students should use the properties of equality to discuss why the process of elimination maintain the same solutions.</td>
<td>Students should be able to understand the process of elimination through simple intuitive problems. <strong>Example:</strong> Given that the sum of two numbers is 10 and their difference is 4, what are the numbers? Explain how your answer can be deduced from the fact that the two numbers, ( x ) and ( y ), satisfy the equations ( x + y = 10 ) and ( x - y = 4 ).</td>
</tr>
<tr>
<td><a href="#">September 2017 Update - Feedback</a></td>
<td>NC Math 1 Unit 5 Systems of Equations and Inequalities</td>
</tr>
</tbody>
</table>
When an equation is multiplied by a constant the set of solutions remains the same. Graphically it is the same line.

When two linear equations are added together, a third linear equation is formed that shares a common solution as the original equations. Graphically this means the three linear equations all intersect at the same point.

The process of elimination is to obtain the value for one of the coordinates. Graphically, it is to get either a horizontal or vertical line that goes through the point of intersection.

**Example:** Which of the following systems is equivalent to \( x - 2y = 4 \) and \( 3x + y = 9 \)?

A) \( x - 2y = 4 \) and \( 6x + 2y = 9 \)
B) \( -3x + 6y = 4 \) and \( 3x + y = 9 \)
C) \( x - 2y = 4 \) and \( 6x - 2y = 18 \)
D) \( \frac{1}{2}x - y = 2 \) and \( 3x + y = 9 \)

NC.M1.A.REI.6
Solve systems of equations.
Use tables, graphs, or algebraic methods (substitution and elimination) to find approximate or exact solutions to systems of linear equations and interpret solutions in terms of a context.

### Concepts and Skills

**Pre-requisite**
- Analyze and solve pairs of simultaneous linear equations by graphing and substitution (8.EE.8)
- Create equations for systems of equations (NC.M1.A.CED.3)
- Justify the steps in a solving process (NC.M1.A.REI.1)
- Solve linear equations in one variable (NC.M1.A.REI.3)
- Understand the mathematical reasoning behind the process of elimination (NC.M1.A.REI.5)
- Understand every point on a graph is a solution to its associated equation (NC.M1.A.REI.10)

### Connections
- Understand the mathematical reasoning behind the methods of graphing, using tables and technology to solve systems and equations (NC.M1.A.REI.11)
- Analyze linear functions (NC.M1.F-IF.7)

### The Standards for Mathematical Practices
Implementing the Standards for Mathematical Practice

**Connections**

*Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.*

**Disciplinary Literacy**

*As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.*

Students should be able to discuss their solution method and the steps in the solving process.
New Vocabulary: elimination

Students solve a system of equations and then interpret its solution. Students should be able to solve a system from a contextual situation. Therefore, this standard should be taught with NC.M1.A-CED.3.

Students should not be required to use one method over another when solving a system of equations, but should be allowed to choose the best option for the given scenario. The focus of this standard should also not be limited to the algebraic methods.

Students were taught substitution and graphing methods in 8th grade.

This is a capstone standard supported by several standards in this course. In order to have a complete understanding of this standard, these standards must be incorporated.
- The ability to create equations for a system from a contextual situation is addressed in NC.M1.A-CED.3.
- The understanding of the elimination method is addressed NC.M1.A-REI.5.
- The understanding for using methods graphing, and tables is taught in NC.M1.A-REI.11.

Include cases where the two equations describe the same line (yielding infinitely many solutions) and cases where two equations describe parallel lines (yielding no solution); connect to NC.M1.G-

Example: José had 4 times as many trading cards as Philippe. After José gave away 50 cards to his little brother and Philippe gave 5 cards to his friend for his birthday, they each had an equal amount of cards. Write a system to describe the situation and solve the system.

Example: A restaurant serves a vegetarian and a chicken lunch special each day. Each vegetarian special is the same price. Each chicken special is the same price. However, the price of the vegetarian special is different from the price of the chicken special. What is the cost of each lunch special?
- On Thursday, the restaurant collected $467 selling 21 vegetarian specials and 40 chicken specials.
- On Friday, the restaurant collected $484 selling 28 vegetarian specials and 36 chicken specials.

Example: The math club sells candy bars and drinks during football games. How much does each candy bar sell for?
- 60 candy bars and 110 drinks will sell for $265.
- 120 candy bars and 90 drinks will sell for $270.

Example: Two times Antonio’s age plus three times Sarah’s age equals 34. Sarah’s age is also five times Antonio’s age. How old is Sarah?

Example: Lucy and Barbara began saving money the same week. The table below shows the models for the amount of money Lucy and Barbara had saved after x weeks.

<table>
<thead>
<tr>
<th>Lucy’s Savings</th>
<th>f(x) = 10x + 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Barbara’s Savings</td>
<td>g(x) = 7.5x + 25</td>
</tr>
</tbody>
</table>

After how many weeks will Lucy and Barbara have the same amount of money saved?

Example: A streaming movie service has three monthly plans to rent movies online. Graph the equation of each plan and analyze the change as the number of rentals increase. When is it beneficial to enroll in each of the plans?
- Basic Plan: $3 per movie rental
- Watchers Plan: $7 fee + $2 per movie with the first two movies included with the fee
- Home Theater Plan: $12 fee + $1 per movie with the first four movies included with the fee
GPE.5, which requires students to prove the slope criteria for parallel lines.
NC.M1.A-REI.11  
**Represent and solve equations and inequalities graphically**

Build an understanding of why the $x$-coordinates of the points where the graphs of two linear, exponential, or quadratic equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$ and approximate solutions using a graphing technology or successive approximations with a table of values.

### Concepts and Skills

**Pre-requisite**
- Solving multi-step linear equations (8.EE.7)
- Analyze and solve pairs of simultaneous linear equations by graphing and substitution (8.EE.8)
- Understand every point on a graph is a solution to its associated equation (NC.M1.A-REI.10)

**Connections**
- Creating and solving one variable equations and systems of equations (NC.M1.A-CED.1, NC.M1.A-CED.3)
- Solving systems of equations (NC.M1.A-REI.6)

### The Standards for Mathematical Practices

#### Connections

*Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.*

4 – Model with mathematics

#### Disciplinary Literacy

*As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.*

New Vocabulary: exponential function, quadratic function

### Mastering the Standard

#### Comprehending the Standard

For a complete understanding of the standard, students will need exposure to both parts of this standard.

First, students should be able to see the connection between graphs and tables of two functions, the points they have in common and the truthfulness of the equation.

**Example:**

For example:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x) = 2x - 4$</th>
<th>$x$</th>
<th>$g(x) = \frac{1}{2}x + \frac{1}{2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-4</td>
<td>0</td>
<td>.5</td>
</tr>
<tr>
<td>1</td>
<td>-2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>2</td>
<td>1.5</td>
</tr>
</tbody>
</table>

#### Assessing for Understanding

**Example:** The functions $f(m) = 18 + 0.4m$ and $g(m) = 11.2 + 0.54m$ give the lengths of two different springs in centimeters, as mass is added in grams, $m$, to each separately.

a) Graph each equation on the same set of axes.

b) What mass makes the springs the same length?

c) What is the length at that mass?

d) Write a sentence comparing the two springs.

**Example:** Solve the following equations by graphing. Give your answer to the nearest tenth.

a) $3(2^x) = 6x - 7$

b) $10x + 5 = -x + 8$
Because \( f(x) = g(x) \) when \( x = 3 \), 3 is the solution to the equation \( 2x - 4 = \frac{1}{2}x + \frac{1}{2} \).

(As an extension, students could write an inequality to describe the relationship between the functions when \( x < 3 \) and when \( x > 3 \).)

In Math 1, students are expected to solve linear systems of equations algebraically. All other systems should be solved with technology, tables, and graphs.

Second, students should be able to use a system of equations to solve systems of equations.

For example:

Solve: \( 3x^2 - 2x + 1 = \frac{1}{2}x + 5 \)

Rewrite the equations as a system of equations

\[
\begin{cases}
  f(x) = 3x^2 - 2x + 1 \\
  g(x) = \frac{1}{2}x + 5
\end{cases}
\]

Using technology, graph the equations and look for point of intersection, where the same \( x \) produces \( f(x) = g(x) \).

In Math 1, students are expected to solve linear equations and quadratic equations with square roots and factoring. In all other equations, such as exponential equations, solutions should be approximated with technology, tables and graphs.

---

**Example:** The population of a country is initially 2 million people and is increasing at 4% per year. The country’s annual food supply is initially adequate for 4 million people and is increasing at a constant rate adequate for an additional 0.5 million people per year.

a) Based on these assumptions, in approximately what year will this country first experience shortages of food?

b) If the country doubled its initial food supply and maintained a constant rate of increase in the supply adequate for an additional 0.5 million people per year, would shortages still occur? In approximately which year?

c) If the country doubled the rate at which its food supply increases, in addition to doubling its initial food supply, would shortages still occur?

(https://www.illustrativemathematics.org/content-standards/HSA/REI/D/11/tasks/645)
NC.M1.A-REI.12

*Represent and solve equations and inequalities graphically*

Represent the solutions of a linear inequality or a system of linear inequalities graphically as a region of the plane.

<table>
<thead>
<tr>
<th>Concepts and Skills</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pre-requisite</strong></td>
</tr>
<tr>
<td>● Solve two-step linear inequalities (7.EE.4b)</td>
</tr>
<tr>
<td>● Solve linear inequalities in one variable (NC.M1.A-REI.3)</td>
</tr>
<tr>
<td>● Understand every point on a graph is a solution to its associated equation (NC.M1.A-REI.10)</td>
</tr>
<tr>
<td><strong>Connections</strong></td>
</tr>
<tr>
<td>● Create one variable linear inequalities and use the inequality to solve problems (NC.M1.A-CED.1)</td>
</tr>
<tr>
<td>● Create a system of linear inequalities to model a situation in context (NC.M1.A-CED.3)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>The Standards for Mathematical Practices</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Implementing the Standards for Mathematical Practice</strong></td>
</tr>
<tr>
<td><strong>Connections</strong></td>
</tr>
<tr>
<td>Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.</td>
</tr>
<tr>
<td><strong>Disciplinary Literacy</strong></td>
</tr>
<tr>
<td>As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.</td>
</tr>
<tr>
<td>Students should be able to explain the reasoning behind their graphical representation of an inequality or system of inequalities.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mastering the Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Comprehending the Standard</strong></td>
</tr>
<tr>
<td>Students should understand that since there is no way to list every solution to a linear inequality in two variables, the solutions must be represented graphically.</td>
</tr>
<tr>
<td>It is an American tradition to shade the region that represent the solutions of the inequality. In other countries, they shade regions of the plane that do not contain solutions, marking that region out. This results in an unmarked solution region making it easier to identify and work with points in the solution region. This means that it is important for students to understand what the shaded region represents according to the context of the problem.</td>
</tr>
<tr>
<td><strong>Assessing for Understanding</strong></td>
</tr>
<tr>
<td>Students should be able to represent solutions to linear inequalities and systems of linear inequalities as a region of a plane.</td>
</tr>
<tr>
<td><strong>Example:</strong> Graph the solution set for the following system of inequalities:</td>
</tr>
<tr>
<td>[ 3x + 5y \leq 10 ] [ y &gt; -4 ]</td>
</tr>
<tr>
<td><strong>Example:</strong> Graph the system of linear inequalities below and determine if (3, 2) is a solution to the system.</td>
</tr>
<tr>
<td>[ x - 3y &gt; 0 ] [ x + y \leq 2 ] [ x + 3y &gt; -3 ]</td>
</tr>
<tr>
<td><strong>Example:</strong> Graph the following inequalities to determine the solution to the system:</td>
</tr>
<tr>
<td>[ 3x - 4y \leq 7 ] [ y &gt; -2x + 6 ] [ -9x + 4y \geq 1 ]</td>
</tr>
</tbody>
</table>
**Example:** Given below are the graphs of two lines, \( y = -0.5x + 5 \) and \( y = -1.25x + 8 \), and several regions and points are shown. Note that \( C \) is the region that appears completely white in the graph.

a) For each region and each point, write a system of equations or inequalities, using the given two lines, that has the region or point as its solution set and explain the choice of \( \leq, \geq, \) or \( = \) in each case. (You may assume that the line is part of each region.)

b) The coordinates of a point within a region have to satisfy the corresponding system of inequalities. Verify this by picking a specific point in each region and showing that the coordinates of this point satisfy the corresponding system of inequalities for that region.

c) In the previous part, we checked that specific coordinate points satisfied our inequalities for each region. Without picking any specific numbers, use the same idea to explain how you know that all points in the 3rd quadrant must satisfy the inequalities for region A.

(https://www.illustrativemathematics.org/content-standards/HSA/REI/D/12/tasks/1205)
The North Carolina High School Collaborative Instructional Framework

NC Math 1
Unit 6: Descriptive Statistics

9 Days Block Schedule
18 Days Traditional Schedule

RESEARCH BRIEF: Univariate Statistics

Essential Questions:
● When is it most effective to use a box plot vs a histogram?
● How are shape, center, and spread used to interpret differences between data sets?
● How do extreme data points affect the shape, center, and spread of data sets?

Learning Outcomes
● Students will interpret data using box plots and histograms.
● Students will compare the shape, center, and spread of different data sets.
● Students will determine the effects of outliers on the shape, center, and spread of data sets.

Student Objectives
● I will use histograms and box plots to interpret data. NC.M1.S-ID.1
● I will compare the shape, center, and spread of different data sets. NC.M1.S-ID.2
● I will explain the effects of extreme data points (outliers) on shape, center, and/or spread. NC.M1.S-ID.3

Standards Addressed in this Unit
Understand how to summarize, represent, interpret and compare data on a single count or measurement variable.
<table>
<thead>
<tr>
<th>OCS Priority Standard(s)</th>
<th>Supporting Standard(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>NC.M1.S-ID.3</strong>: Summarize, represent, and interpret data on a single count or measurement variable. Examine the effects of extreme data points (outliers) on shape, center, and/or spread.</td>
<td><strong>NC.M1.S-ID.1</strong>: Summarize, represent, and interpret data on a single count or measurement variable. Use technology to represent data with plots on the real number line (histograms and box plots). <strong>NC.M1.S-ID.2</strong>: Summarize, represent, and interpret data on a single count or measurement variable. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. Interpret differences in shape, center, and spread in the context of the data sets.</td>
</tr>
</tbody>
</table>
NC.M1.S-ID.1
*Summarize, represent, and interpret data on a single count or measurement variable.*
Use technology to represent data with plots on the real number line (histograms and box plots).

### Pre-requisite
- Displaying numerical data on line plots, dot plots, histograms and dot plots (6.SP.4)

### Connections
- Comparing two or more data distributions using shape and summary statistics (NC.M1.S-ID.2)
- Examining the effects of outliers on the shape, center, and/or spread of data (NC.M1.S-ID.3)

### Concepts and Skills

<table>
<thead>
<tr>
<th>School</th>
<th>Class period (minutes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lincoln Middle</td>
<td>45</td>
</tr>
<tr>
<td>Central Middle</td>
<td>65</td>
</tr>
<tr>
<td>Oak Grove Middle</td>
<td>70</td>
</tr>
<tr>
<td>New Hope Middle</td>
<td>55</td>
</tr>
<tr>
<td>Sunnyside Middle</td>
<td>50</td>
</tr>
<tr>
<td>Pine Grove Middle</td>
<td>60</td>
</tr>
</tbody>
</table>
Summary statistics include:
- 5-Number summary: minimum value (minX), maximum value (maxX), median (Med), lower quartile (Q₁) and upper quartile (Q₃)
- mean (\( \bar{x} \))
- Sum (\( \sum x \))
- standard deviation (\( Sx \))*

Graphs include:
- Histograms
- Modified Box plots – plots outliers as individual points. A point is determined to be an outlier if:
  - Lower outlier(s) < 1.5 \cdot IQR
  - Upper outlier(s) > 1.5 \cdot IQR

*While technology gives values for the population standard deviation (\( \sigma x \)), students will not use this measurement at this level.

<table>
<thead>
<tr>
<th>School</th>
<th>Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fairview Middle</td>
<td>55</td>
</tr>
<tr>
<td>Jefferson Middle</td>
<td>60</td>
</tr>
<tr>
<td>Roosevelt Middle</td>
<td>60</td>
</tr>
<tr>
<td>Green Middle</td>
<td>65</td>
</tr>
<tr>
<td>Hope Middle</td>
<td>55</td>
</tr>
</tbody>
</table>

**Example:** The following data set shows the number of songs downloaded in one week by each student in Mrs. Jones class: 10, 20, 12, 14, 12, 27, 88, 2, 7, 30, 16, 16, 32, 25, 15, 4, 0, 15, 6, 1, 0, 15, 12, 10, and 7.

a) What are the summary statistics for the data?
b) Construct two different graphs of the data.
c) Describe the distribution of the data, citing both of the plots and the numerical summary statistics.
d) What are the advantages to each data display? Explain.
NC.M1.S-ID.2

Summarize, represent, and interpret data on a single count or measurement variable.

Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. Interpret differences in shape, center, and spread in the context of the data sets.

Pre-requisite

- Relating the choice of center and variability to shape of data (6.SP.5d)
- Informally compare graphical displays of two distributions to make inferences about two populations (7.SP.3)
- Informally compare numerical summaries of two distributions to make inferences about two populations (7.SP.4)
- Use technology to represent data (NC.M1.S-ID.1)

Connections

- Effects of outliers on shape, center, and/or spread (NC.M1.S-ID.3)

Concepts and Skills

**The Standards for Mathematical Practices**

Implementing the Standards for Mathematical Practice

**Connections**

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics
5 – Use appropriate tools strategically
6 – Attend to precision

**Vocabulary**

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication. The following vocabulary is new to this course and supported by this standard:

New Vocabulary: standard deviation, outlier

Mastering the Standard

<table>
<thead>
<tr>
<th>Comprehending the Standard</th>
<th>Assessing for Understanding</th>
</tr>
</thead>
<tbody>
<tr>
<td>In middle school, students related the measure of center and variability to the shape and context of the data. Students know that symmetrical displays are more appropriate for the mean as a measure of center and mean absolute deviation (M.A.D) as a measure of variability. Likewise, they understand that skewed distributions or distributions with outliers are better described using median as a measure of center due to the fact that it is a resistant measure of center; and the interquartile range (IQR) as a measure of</td>
<td></td>
</tr>
</tbody>
</table>
| Given two or more sets of data, students compare datasets and identify similarities and differences in shape, center and spread within the context of the data.  
**Example:** Ms. Williams wants to analyze the scores for the first unit test of her 1st period and 4th period NC Math 1 classes. The scores for each class are below.  
**1st Period:**  
82, 100, 94, 68, 34, 72, 70, 96, 99, 92,  
90, 85, 70, 46, 71, 71, 77, 78, 95, 82,  
80, 100, 99, 72, 69, 74, 84, 87  
**4th Period:**  
100, 95, 72, 80, 97, 78, 89, 100, 93, 95, 66,  
87, 85, 98, 89, 86, 80, 79, 94, 90, 92, 87, 88  
81, 82  
a) Calculate the mean, median, standard deviation, and interquartile range for each class.  
b) Construct an appropriate graph to compare the two classes. |
variability.

Context also plays an important role in the choice of summary statistic utilized. Students can examine the context to rationalize why particular measures are more appropriate than others.

The standard deviation is a new summary statistic for students. Its development should be based on the M.A.D (Mean Absolute Deviation) learned in the 7th grade. Essentially, students need to understand that SD like M.A.D is a measure of variability in the data. The larger SD, the more variable the data. Students should also know that standard deviation allows comparison of variability in multiple data sets regardless of the unit of measurement for the data sets.

An understanding of how the standard deviation is calculated can help students to conceptualize the value and why it’s primarily used in association with mean as a measure of center.

\[ S_x = \sqrt{\frac{1}{n-1} \sum (x_i - \bar{x})^2} \]

Using a relatively smaller data set and the list feature in the graphing calculator can make the calculations easier during development of the concept.

c) Write several sentences to compare the class grades in context.

Given two or more graphs, students compare datasets and identify similarities and differences in shape, center and spread within the context of the data.

Example: Delia wanted to find the best type of fertilizer for her tomato plants. She purchased three types of fertilizer and used each on a set of seedlings. After 15 days, she measured the heights (in cm) of each set of seedlings. The data she collected and plots are shown below. Write a brief description comparing the three types of fertilizer. Which fertilizer do you recommend that Delia use? Explain your answer.

<table>
<thead>
<tr>
<th>Fertilizer A</th>
<th>Fertilizer B</th>
<th>Fertilizer C</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.1</td>
<td>11.0</td>
<td>10.5</td>
</tr>
<tr>
<td>5.0</td>
<td>9.2</td>
<td>11.8</td>
</tr>
<tr>
<td>3.2</td>
<td>10.5</td>
<td>15.5</td>
</tr>
<tr>
<td>5.5</td>
<td>6.3</td>
<td>10.5</td>
</tr>
<tr>
<td>6.2</td>
<td>17.0</td>
<td>9.5</td>
</tr>
</tbody>
</table>

Online Tools
NC.M1.S-ID.3
Summarize, represent, and interpret data on a single count or measurement variable.
Examine the effects of extreme data points (outliers) on shape, center, and/or spread.

<table>
<thead>
<tr>
<th>Concepts and Skills</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pre-requisite</strong></td>
</tr>
<tr>
<td>● Describing striking deviations from the overall pattern of a distribution (6.SP.5c)</td>
</tr>
<tr>
<td>● Use technology to create boxplots and histograms (NC.M1.S-ID.1)</td>
</tr>
<tr>
<td><strong>Connections</strong></td>
</tr>
<tr>
<td>● Comparing two or more data distributions using shape and summary statistics (NC.M1.S-ID.2)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>The Standards for Mathematical Practices</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Implementing the Standards for Mathematical Practice</strong></td>
</tr>
<tr>
<td><strong>Connections</strong></td>
</tr>
<tr>
<td>Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.</td>
</tr>
<tr>
<td>3 – Construct a viable argument and critique the reasoning of others</td>
</tr>
<tr>
<td>4 – Model with mathematics</td>
</tr>
<tr>
<td>5 – Use appropriate tools strategically</td>
</tr>
<tr>
<td>6 – Attend to precision</td>
</tr>
<tr>
<td><strong>Vocabulary</strong></td>
</tr>
<tr>
<td>As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication. The following vocabulary is new to this course and supported by this standard:</td>
</tr>
<tr>
<td>New Vocabulary: outlier, standard deviation</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mastering the Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Comprehending the Standard</strong></td>
</tr>
<tr>
<td>An important part of data analysis includes examining data for values that represent abnormalities in the data. In MS, students informally addressed “striking deviations from the overall pattern” of a data distribution. The identification of outliers is formalized in this standard. A value is mathematically determined to be an outlier if the value falls 1.5 IQRs below the 1st quartile or above the third quartile in a data set.</td>
</tr>
<tr>
<td>● Lower outlier(s) &lt; 1.5 \cdot \text{IQR}</td>
</tr>
<tr>
<td>● Upper outlier(s) &gt; 1.5 \cdot \text{IQR}</td>
</tr>
<tr>
<td><strong>Assessing for Understanding</strong></td>
</tr>
<tr>
<td>Students understand and use the context of the data to explain why its distribution takes on a particular shape (e.g. Why is the data skewed? Are there outliers?)</td>
</tr>
<tr>
<td><strong>Example:</strong></td>
</tr>
<tr>
<td>Why does the shape of the distribution of incomes for professional athletes tend to be skewed to the right? Why does the shape of the distribution of test scores on a really easy test tend to be skewed to the left? Why does the shape of the distribution of heights of the students at your school tend to be symmetrical?</td>
</tr>
<tr>
<td>Students should identify outliers of the data set and determine the effect outliers will have on the shape, center, and spread of a data set.</td>
</tr>
<tr>
<td><strong>Example:</strong> The heights of players on the Washington High School’s Girls basketball team are recorded below:</td>
</tr>
<tr>
<td>5’ 10”</td>
</tr>
</tbody>
</table>
The mean and standard deviation are most commonly used to describe sets of data. However, if the distribution is extremely skewed and/or has outliers, it is best to use the median and the interquartile range to describe the distribution since these measures are not sensitive to outliers.

It is important to detect outliers within a distribution, because they can alter the results of the data analysis. The mean is more sensitive to the existence of outliers than other measures of center.

A student transfers to Washington High and joins the basketball team. Her height is 6’2”

a) What is the mean height of the team before the new player transfers in? What is the median height?

b) What is the mean height after the new player transfers? What is the median height?

c) What affect does her height have on the team’s height distribution and stats (center and spread)?

d) Which measure of center most accurately describes the team’s average height? Explain.

Example: The table to the left shows the length of a class period for each of the school’s listed. If Cherry Lane Middle School’s class period length of 100 minutes is added to the data above, what effect will it have on the mean, median, interquartile range, standard deviation and on the graph of the data?
<table>
<thead>
<tr>
<th>Revised Bloom’s Taxonomy</th>
<th>Webb’s DOK Level 1 Recall &amp; Reproduction</th>
<th>Webb’s DOK Level 2 Skills &amp; Concepts</th>
<th>Webb’s DOK Level 3 Strategic Thinking/Reasoning</th>
<th>Webb’s DOK Level 4 Extended Thinking</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Remember</strong></td>
<td>Recall, observe, &amp; recognize facts, principles, properties</td>
<td>Specify and explain relationships (e.g., non-examples/examples; cause-effect)</td>
<td>Use concepts to solve non-routine problems</td>
<td>Relate mathematical or scientific concepts to other content areas, other domains, or other concepts</td>
</tr>
<tr>
<td></td>
<td>Recall/identify conversions among representations or numbers (e.g., customary and metric measures)</td>
<td>Make and record observations</td>
<td>Explain, generalize, or connect ideas using supporting evidence</td>
<td>Develop generalizations of the results obtained and the strategies used (from investigation or readings) and apply them to new problem situations</td>
</tr>
<tr>
<td><strong>Understand</strong></td>
<td>Evaluate an expression</td>
<td>Explain steps followed</td>
<td>Use different solutions or approaches possible</td>
<td>Select or devise approach among many alternatives to solve a problem</td>
</tr>
<tr>
<td></td>
<td>Locate points on a grid or number on number line</td>
<td>Summarize results or concepts</td>
<td>Explain thinking/reasoning when more than one solution or approach is possible</td>
<td>Conduct a project that specifies a problem, identifies solution paths, solves the problem, and reports results</td>
</tr>
<tr>
<td></td>
<td>Solve a one-step problem</td>
<td>Make basic inferences or logical predictions from data/observations</td>
<td>Explain phenomena in terms of concepts</td>
<td>Evaluate and develop a logical argument for concepts or solutions</td>
</tr>
<tr>
<td></td>
<td>Represent math relationships in words, pictures, or symbols</td>
<td>Use models/diagrams to represent or explain mathematical concepts</td>
<td>Use evidence to support claims or statistical models</td>
<td>Gather, analyze, &amp; evaluate information to draw conclusions</td>
</tr>
<tr>
<td></td>
<td>Read, write, compare decimals in scientific notation</td>
<td>Make and explain estimates</td>
<td>Make and justify conjectures</td>
<td>Apply understanding in a novel way, provide argument or justification for the application</td>
</tr>
<tr>
<td><strong>Apply</strong></td>
<td>Follow simple procedures (recipe-type directions)</td>
<td>Select a procedure according to criteria and perform it</td>
<td>Design investigation for a specific purpose or research question</td>
<td>Analyze multiple sources of evidence</td>
</tr>
<tr>
<td></td>
<td>Calculate, measure, apply a rule (e.g., area, perimeter)</td>
<td>Solve routine problem applying multiple concepts or decision points</td>
<td>Conduct a designed investigation</td>
<td>Analyze complex/abstract themes</td>
</tr>
<tr>
<td></td>
<td>Apply algorithm or formula (e.g., area, perimeter)</td>
<td>Retrieve information from a table, graph, or figure and use it to solve a problem requiring multiple steps</td>
<td>Use concepts to solve non-routine problems</td>
<td>Gather, analyze, and evaluate information</td>
</tr>
<tr>
<td></td>
<td>Solve linear equations</td>
<td>Translate between tables, graphs, words, and symbolic notations (e.g., graph data from a table)</td>
<td>Use &amp; show reasoning, planning, and evidence</td>
<td>Analyze and draw conclusions from data, citing evidence</td>
</tr>
<tr>
<td></td>
<td>Make conversions among representations or numbers, or within and between customary and metric measures</td>
<td>Construct models given criteria</td>
<td>Translate between problem &amp; symbolic notation when not a direct translation</td>
<td>Evaluate data from complex graphs</td>
</tr>
<tr>
<td><strong>Analyze</strong></td>
<td>Retrieve information from a table or graph to answer a question</td>
<td>Categorize, classify materials, data, figures based on characteristics</td>
<td>Compare information within or across data sets or texts</td>
<td>Analyze similarities/differences between procedures or solutions</td>
</tr>
<tr>
<td></td>
<td>Identify whether specific information is contained in graphic representations (e.g., table, graph, T-chart, diagram)</td>
<td>Organize or order data</td>
<td>Analyze and draw conclusions from data, citing evidence</td>
<td>Analyze model</td>
</tr>
</tbody>
</table>